KONINKLIJKE AKADEMIE VAN WETENSCHAPPEN
TE AMSTERDAM

PROCEEDINGS

VOLUME XXXVI

No. 3

President: J. VAN DER HOEVE
Secretary: B. BROUWER

CONTENTS

Physiology. — *Diet and reproduction IV. The bipartite nature of vitamin-E.* (A provisional communication). By G. GRIJNS and Miss E. DINGEMANSE.

(Communicated at the meeting of March 25, 1933).

In 1928 GRIJNS, DE HAAN en VAN DER LOEFF [1]) came to the conclusion that for production of fertilising spermatozoids and for normal gestation different vitamins are responsible, while for milksecretion it seemed dubious whether a vitamin or an albuminfraction is the controling factor.

On experiments about milksecretion one of us will report elsewhere.

According to most investigators wheat germ oil promotes fertility in males and in females and these two functions are attributed to the same vitamin. From our point of view it seemed desirable to try the separation of these two.

In a conference in the Pharmaco-Therapeutical laboratory at Amsterdam with Professor Dr. E. LAQUEUR we decided to take up the investigation together. Miss DINGEMANSE should do the chemical work and GRIJNS the biological.

On the ration XI with cod liver oil the growth rate of our rats was less than that of the stock rats. So we tried two rations, we adopted from American publications, viz.

Ration LVII.		Ration LVIII.	
Casein	70	Skimmed milkpowder	250
Amylum oryzae	100	Amylum oryzae	50
Hydrogenated fat	40	Dextrin	110
Butterfat	15	Marmite	25
Marmite	20	Ferric citrate	1
Decitrated lemonsqash	25	Decitrated lemonsquash	25
Ostelin oil (1/10)	2	Hydrogenated fat	50
Salt (Osborn mixture)	20		

[1]) GRIJNS G., DE HAAN K. and LOEFF, J. A. VAN DER: Diet and reproduction III, Proceedings Vol. XXXI, 7, p. 721, 1928.

3 males and 6 females on ration LVII proved all fertile and had young twice or more. They were almost unable to raise their youngs, seeing that only 5 of 61 were weaned.

On ration LVIII we put 6 males and 12 females immediately after weaning. They thrived fairly well and when mated all proved fertile. They bore 130 young in 23 litters, 50 of those were weaned.

From these 20 females and 12 males were kept on ration LVIII.
When they had reached 160—180 G. weight they were mated. 18 females had young within 2 months; one had died before and one had been inseminated, but not shown EVANS placenta sign. In 21 litters they had 137 Young from which 84 were weaned.

For testing the males we every morning microscopised vaginal smears of a lot of stock rats and chose those in the first stage of oestrus, to mate them with the males. Next morning they were controled for insemination and when heat was over, replaced by a new one. If 5 or 6 of such meetings did not lead to an insemination the male was considered sterile.

From the 12 males 4 showed impotency. These were treated with 0.25 G. of a wheat germ oil., that had been stored for about a year in our laboratory. They received it daily during 3 weeks and then were tested again. 3 inseminated their females, 1 not.

The oil proved to have retained its activity.

We now continued our experiment with the third generation on ration LVIII. 24 out of 27 females had young in the first mating, the 3 remaining in a second. We see from this, that in skimmed milkpowder (Nutricia Ltd.) there is abundant female fertility vitamin.

From 18 males only 5 were fertile. Here again the difference between the two sexes. The unfertile 13 were divided in 2 groups. 3 animals got a supply of wheat germ oil of 0.25 G. a day during 3 weeks the 10 other ones a preparation, prepared in the following way. Wheatgerm was, according to the description of EVANS in "The antisterility vitamin fat soluble E.", extracted by ether and the oil saponified with alcoholic KOH. The output of unsaponifiable matter was about 9 % of the original oil. This was treated with pentane at a low temperature. From the pentane 2.7 % of dry residue of the original oil was recovered. This was treated with methyl alcohol and after evaporation of the methyl alcohol a thickish dark read oil remained amounting to 0.8 % of dry residue of the original oil.

We used 2 preparations made in this way from two lots of wheat germ. The oil was dissolved in petroleum ether 4 to 10 and from this solution 6 mG. in codliveroil were given daily to the infertile males.

After 3 weeks all were tested again. From the wheatgermoil 2 had become fertile, from the 10 on the extract only 2.

A longer administration of the product to the sterile rats had no effect, even though the dose was increased.

The success is a moderate one. But we see from this experiment that the two rations are unsuitable for our purpose and the method of estimating

the potency of some substance for reproduction in males by feeding it to infertile ones is dissatisfying.

As one of us has shown before, males on a ration of maize and peanutmeal are as a rule unfertile in the second generation, but when they get stockration from weaning they are fertile. In the second generation kept on maize ration sooner or later the germinative epithelium in the testes degenerates and infertility soon becomes irreparable. Now we have no means to estimate during life the degree to which the epithelium is damaged; one might remove one testicle for microscopical examination, but it is not sure, that the other gonad will suffer to the same degree, — and such control can but once be done. While trying to establish infertility it may become irreparable and an active preparation seem inactive.

EVANS also noted, that only $^1/_5$ of the infertile males can be restored to fertility by vitamin E.

For these reasons we considered the preparations to be active notwithstanding the small effect, we had observed, (also regarding the positive effect of EVANS analogous extract).

We decided to return to our old ration XI and to maizefood I. We now tried the following 3 preparations:

E. 3.

This was prepared according to SCHERING-KAHLBAUM's patent for extraction of female hormon from wheat.

10 K.G. wheatgerm were stirred with 15 K.G. of calcium hydroxide, 15 K.G. of Kaolin and 50 K.G. of alcohol 40 % for 3 hours at 60° C.; filtered and the filtrate saturated with CO_2; the alcohol evaporated and the residue exhausted with ether. The residue of the ether extract is once more subjected to the same manipulations. The resulting residue, 1.2 G. is dissolved in petroleum ether and repeatedly shaken with 92 % alcohol. The collected alcohol is shaken twice with petroleum ether and the petroleum ether solutions combined, evaporated in vacuo till dryness. The output 1.07 G. was dissolved in 20 cc. of petroleum ether and labeled E. 3.

Two more preparations were obtained from an extract, as had been tested in the experiments already described.

E. 4. A.

8 G. of a methyl alcoholic extract prepared by the method of EVANS were solved in petroleum ether and 5 times shaken with 90 % methyl alcohol, the methyl alcohol 3 times with petroleum ether, the etheric solutions joined and evaporated, at low temperature: dry residue 5.3 G. This was dissolved in benzene and shaken with 70 % alcohol, wherein 2.65 G. KOH had been dissolved. The alcoholic solution was twice shaken with benzene and the benzenic solution with 70 % alcohol. The combined benzene solutions 3 times washed with acidulated water, then with water until the reaction became neutral and then evaporated. The dry residue, 1.38 G. is solved in 20 cc. of petroleum ether as E. 4. A.

E. 4. B.

The collected alcoholic solutions of the former preparation were partly evaporated in vacuo, the residue extracted with benzene and washed also with acidulated water and with water till neutral reaction. The residue of this fraction is 397 mG. It is solved in 20 cc. petroleum ether and labeled E. 4. B.

We now elaborated the following scheme for our investigation. Young males from the same litter, whose parents had been fed maize ration I, were divided in two lots after weaning, the one receiving the same ration the others in addition to it a quantity of the extract to be tested. We diluted the preparations with cod liver oil to such extent that 0.2 or 0.3 cc. of the oil contained the quantity to be administered. It was given per os by a syringe with a blunt needle.

After attaining maturity the males were placed in separate cages with two females, these were examined daily by vaginal smears on insemination and oestrus. After about 5 weeks the males were killed, the testicles weighed and the epididymis examined for spermatozoids.

The females were tested as before. They received the test oil but after they were mated.

As the extracts we were testing are not definite ones, but only steps on the way to purer substances, we did not use more rats than necessary to show us in which fraction the active compounds are present.

To 4 females we gave 2 times a week 0.045 cc. of extract E. 3. They had shown before resorption of foetus as described by EVANS. They were inseminated, showed red bloodcorpuscles in the vaginal smear in due time and then lost weight and after some weeks oestrus reappeared. 3 rats, sterile according EVANS method, were fed twice a week 0.2, later 0.4 of a dilution $^1/_{100}$ of preparation E. 4. A corresponding to 0.55 mG. of dry residue a week. They all had insemination, placentar sign and resorption of embryos once or twice; no youngs.

Preparation E. 4. B was given to two rats, which have shown resorption on E. 3. They received 0.4 cc. of an $^1/_{100}$ dilution or 0.165 mG. a week from 6—6. The first was inseminated on 7—6; had red corpuscles on 20—6 and 6 youngs on 1—7. The other was inseminated on 8—6 and had 10 youngs on 5—7.

One rat that had been sterile on E. 4. A. got 4 youngs when it was given E. 4. B.

Some other rats on E. 4. B. died during observation. One of the difficulties of these experiments is, that the lack of vitamin-E not only prevents reproduction, but it also reduces growth and damages health.

Till 120 or 140 G. female rats will grow fairly well on my XI with codliver oil diet, but then most of them slacken in growth or stop it altogether. They show a more or less miserable aspect, often become bald and are soon old.

The males gave quite other results. We collected our observations in

TABLE I. E.3.

	Preparation				Control			
Litter	N^0.	Insemination	Weight testes	Spermatozoids	N^0.	Insemination	Weight testes	Spermatozoids
2954					3068	+ –	2.40	+ + +
2737	3071	+ +	1.69	+	3070	– –	1.20	–
	3073	+ +	1.36	–	3074	– –	1.00	–
	3076	+ +	1.05	+	3075	+ +	1.50	–
2930	3079	+ +	1.10	+ + +				
	3080	– +	1.03	+				
2941					3081	– –	0.95	–
					3082	– –	1.00	–
2944	3086	+ –	2.40	+ + +	3087	– –	1.87	–
	3088	+ –	1.67	+	3112	+ +	1.10	+ + +
					3115	– –	0.88	–
2951	3223	+ +	1.76	+ + +	3233	– –	1.10	–
	3227	– –	1.11	–				
	Mean		1.46				1.30	

TABLE 2. E.4.A.

	Preparation				Control			
Litter	N^0.	Insemination	Weight testes	Spermatozoids	N^0.	Insemination	Weight testes	Spermatozoids
2948	3137	+ +	3.33	+ + +	3136	+ +	3.18	+ + +
	3139	+ +	3.01	+ + +				
2947	3145	+ +	2.70	+ + +	3148	– –	1.25	–
	3146	– –	1.46	+ + +				
	3149	– –	1.21	+ + +				
2950					3150	– –	0.95	–
2935	3193	+ +	1.73	+ + +	3192	+ +	2.22	+ + +
					3194	– –	1.26	–
3013	3208	– +	1.45	+ + +	3205	– –	1.02	–
	3209	– –	1.25	+ + +	3206	+ +	2.53	+ + +
	Mean		2.02				1.90	

TABLE 3. E.4.B.

Preparation					Control			
Litter	N⁰.	Insemi-nation	Weight testes	Sperma-tozoids	N⁰.	Insemi-nation	Weight testes	Sperma-tozoids
3021	3151	— —	1.95	+				
2981	3154	— —	0.96	—	3155	— —	0.95	—
	3157	— —	0.88	—	3156	— —	1.17	+
	3158	— —	1.17	—				
	3159	— —	1.16	—				
3017	3160	— —	0.88	—	3161	— —	0.95	—
					3162	— —	0.91	—
2935	3191	+ —	1.66	+ + +				
3015	3198	— —	1.02	+ + +	3197	+ +	1.50	+ + +
	3201	+ +	2.68	+ + +	3203	— —	1.10	—
2944	3237	— —	1.24	+				
	3239	— —	1.26	+				
	3241	— —	1.68	+				
	Mean		1.38				1.08	

3 tables and placed experimental rats and controls so, that rats from one litter came in the same line. That makes it easy to find out congenital differences.

The first column gives the serial numbers of the mothers. Under No those of the males are indicated. The columns insemination have two signs, one for each of the two females. In the columns spermatozoids three + signs mean mobile spermatozoids, one + immobile ones and — none found in epididymis.

The weight of the testes is very variable; in the controls we see fluctuations as large as in the experimental groups.

Therefore we think it not justified to draw conclusions from the differences in the three groups.

If we consider the inseminations and the spermatozoids in the epididymis, the E. 3. and the E. 4. A. males are much more fertile than the E. 4. B. rats; especially when we notice that 2 positive males in the E. 4. B. group are from the same litter in which one of the controls was also productive.

We see thus, that the fertilising factor E. for the females is absent in the preparations E. 3. and E. 4. A. present in E. 4. B. The factor E for

the males on the contrary is present in the preparations E. 3. and E. 4. A. absent or at least very scanty in E. 4. B. or alcohol fraction.

The fact, that the purified preparations of EVANS and of SURE were active for males and for females, at first seemed to contradict my assumption of the duplicity of the vitamin E. Now we have been able to separate it in two factors, one of which is effective for females one for males, we may consider the duality proved.

We will go on to purify our products and try to come to pure fractions and then enlarge the number of our experimental rats.

From the Laboratory of animal physiology at Wageningen (Director Dr. G. GRIJNS) *and the pharmaco-therapeutic laboratory at Amsterdam* (Director Prof. Dr. E. LAQUEUR).

Physics. — *Increase of the concentration of H^1H^2 by fractional evaporation and rectification.* By W. H. KEESOM, H. VAN DIJK and J. HAANTJES. (Communication N⁰. 224*a* from the KAMERLINGH ONNES Laboratory at Leiden).

(Communicated at the meeting of March 25, 1933).

Summary. A quantity of 40 liters of liquid hydrogen was reduced to $^1/_2$ liter by fractional evaporation a little above 14° K., and then rectified at 15° K. till a quantity of 4 liters gas was left. This quantity was then reduced to 2 liters by fractional evaporation at 14° K.

The specific weight of the sample obtained proved to be $1.51 \pm 0.05\,\%$ larger than that of ordinary hydrogen. It follows that this hydrogen contains 3 % in volume of H^1H^2.

§ 1. *Introduction.* UREY, BRICKWEDDE and MURPHY [1]) showed that common hydrogen consists of 2 isotopes H^1 and H^2 with masses [2]) which approximately have the ratio 1 : 2. This result was confirmed i.a. by BLEAKNEY [3]) and by KALLMANN and LASAREFF [4]) by means of the mass spectrograph, and by HARDY, BARKER and DENNISON [2]), who investigated the absorption spectra of H^2Cl^{35} and H^2Cl^{37} in the infrared.

The concentration of the hydrogen isotope of mass 2 was put by BLEAKNEY at 1 : 30.000 ± 20 %, whereas HARDY, BARKER and DENNISON found 1 : 35.000 [5]).

1) H. C. UREY, F. G. BRICKWEDDE and G. M. MURPHY, Phys. Rev. (2) **39**, 164, 1932; (2) **40**, 1, 464, 1932.

2) Cf. K. T. BAINBRIDGE, Phys. Rev. (2) **41**, 115, 1932; (2) **42**, 1, 1932. J. D. HARDY E. F. BARKER and D. M. DENNISON, Phys. Rev. (2) **42**, 279, 1932.

3) W. BLEAKNEY. Phys. Rev. (2) **39**, 3, 536; (2) **41**, 32, 1932.

4) H. KALLMANN und W. LASAREFF, Naturwiss. **20**, 206, 1932.

5) The smallness of this concentration explains why O. STERN and M. VOLMER, Ann. d. Phys. (4) **59**, 225, 1919, in their diffusion experiments did not find the hydrogen isotope.

We thought it worth while to try to get a confirmation of those results by preparing a sample of hydrogen in which the presence of the hydrogen isotope could be demonstrated by weighing. Also it seemed important to obtain a quantity, sufficiently large for further investigations, of hydrogen with a known concentration of the isotope.

§ 2. *Method.* As a very effective method for increasing the concentration of the hydrogen isotope fractional distillation presents itself[1]), as a still more effective one we consider rectification in a rectifying column at a temperature near that of the triple point.

Supposing that ordinary hydrogen only consists of the isotopes H^1 and H^2, we may assume that it almost exclusively consists of molecules H^1H^1 and H^1H^2, in the proportion 1 : 16.000.

The ratio of the vapour pressures of solid hydrogen consisting exclusively of H^1H^1 molecules and of solid hydrogen consisting exclusively of H^1H^2 molecules can be derived from theory, and amounts to 2.2[2]) at the triple point. This ratio decreases for increasing temperature. For liquid hydrogen the corresponding ratio is probably smaller than it would be for solid hydrogen at the same temperature. Nevertheless the experiments of UREY and his collaborators and of MEISSNER and STEINER[3]) showed that this ratio also for liquid hydrogen is still appreciable, at least so long as the temperature does not exceed the triple point temperature too much. For making an estimation we assume that for a pressure of some millimeters above the triple point pressure this ratio is 2.0. Then, if by evaporation the volume of the liquid is reduced to the $1/n$th part of this initial value, the concentration of the hydrogen isotope in the remaining liquid will be increased by a factor $n^{\frac{1}{2}}$.

So if we could evaporate 40 l of liquid hydrogen till only 2.5 cm³ was left, the molar concentration of H^1H^2 in the remaining liquid would amount to 0.008.

The increase of the concentration can be made much larger by applying rectification. However, rectifying as large a quantity of liquid hydrogen as mentioned would take very much time. So we chose the following procedure.

We evaporated 10 times a quantity of 4 liters of liquid hydrogen at 14° K till 50 cm³ was left. The gas resulting from this ½ liter of liquid after thorough purification was rectified at 15° K on 4 different days according to the following scheme: (See p. 250)

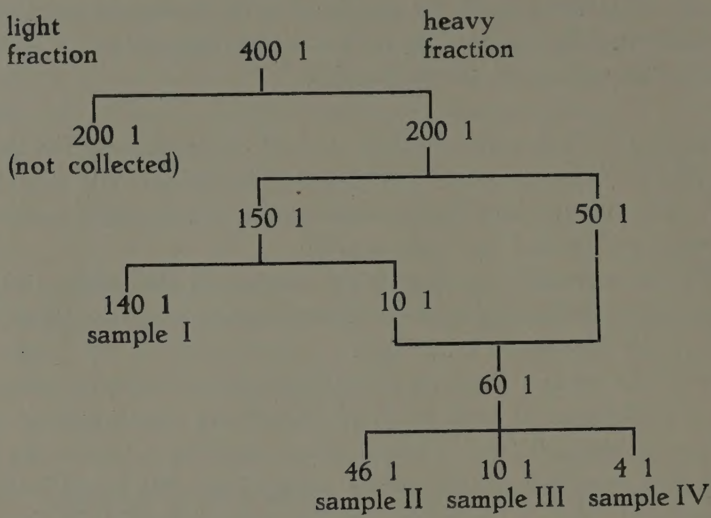
The heavy fraction of 4 liters finally obtained (sample IV) was still divided by liquefaction and fractional evaporation at 14° K into two fractions of 2 liters each: samples V and VI (light and heavy fractions

[1]) For further particulars on the evaporation process at the triple point cf. H. C. UREY, F. G. BRICKWEDDE and G. M. MURPHY, Phys. Rev. (2) **40**, 1, 1932.

[2]) UREY, BRICKWEDDE and MURPHY l.c. in note 1 put this ratio at 2.688. We consider this value to be taken too high.

[3]) W. MEISSNER und K. STEINER, Zs. f. Phys. **79**, 601, 1932.

respectively). For purifying sample VI, we liquefied and evaporated it 6 times. After this we filled a gas density bulb with it.



§ 3. *Apparatus.* The rectification was performed in an apparatus nearly completely made of glass, which is represented in Fig. 1. In the condenser C and the bulb B the pressure was kept at about 60 mm. The condenser could be filled during the process from the bulb B (6 liters). So it was possible to continue the rectification process during 2 hours without interruption.

The proper column contains 11 pans. The still was surrounded by a bath of liquid hydrogen of which the pressure could be regulated. As much as possible column and still are protected against radiation by a metal screen Sc. During the rectification process the reflux ratio was gradually increased from 4 to 20.

Rectification with exhaustion of the lightest fraction was continued till at last only the 3 upper pans contained liquid.

§ 4. *Density measurements.* The density bulb[1]) had a volume of about 760 cm^3. It was filled to a pressure of about 1 atmosphere at 16° C.

The bulb was weighed with ordinary hydrogen first, then with the rectified hydrogen, and finally again with ordinary hydrogen under nearly the same circumstances of temperature and pressure. The difference in weight amounted to about 0.9 mg.

If for normal hydrogen we assume $s_{0,n} = 0.00008987$, the specific weight of the heavy hydrogen (sample VI) was found to be:

calculated from the first weighing of normal hydrogen: $s_{0,h} = 0.00009118$,
,, ,, ,, second ,, ,, ,, ,, ,, $= 0.00009128$.

average $= 0.00009123$.

1) Comp. These Proceedings **34**, 42, 1931. Comm. Leiden N^0. 213*a* § 5.

Hence the difference in specific weight amounts to

$$\frac{s_{0,h} - s_{0,n}}{s_{0,n}} = 0.0151 \pm 0.0005.$$

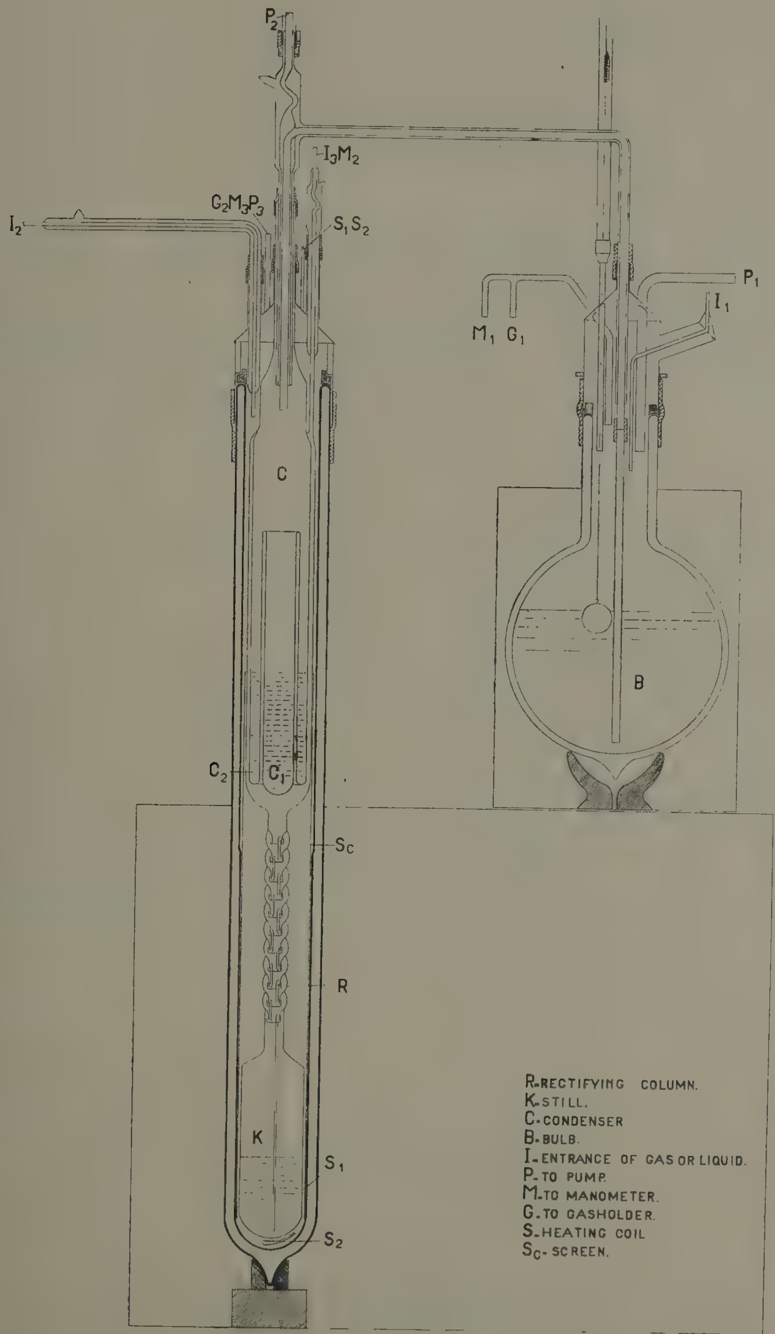


Fig. 1*a*.

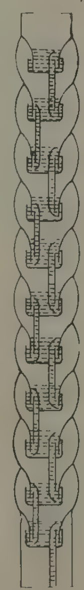


Fig. 1*b*.

It follows that sample VI contains 3 % in volume of H^1H^2 [1]).

§ 5. *Conclusion.* The total reduction in volume of the hydrogen treated amounted to 16.000 : 1, i.e. a reduction 160 : 1 by fractional evaporation, and a reduction 100 : 1 by rectification.

The concentration increase of H^1H^2 amounted to $^3/_{100} : {}^1/_{16000} = 480 : 1$.

If we put the concentration increase by fractional evaporation [2]) $n^{\frac{1}{2}} = 12.6 : 1$, it follows that the concentration increase by rectification was as 38 : 1. As the volume reduction was 100 : 1 as stated, it follows that in the rectification process 38 % of the H^1H^2 molecules remained in the remaining liquid.

The rectification as performed is equivalent to a process in which by slow exhaustion the volume of 400 liters of gas is reduced to 4 liters, the molar concentration of H^1H^2 in the gas exhausted continually being about $^1/_5$ of that in the hydrogen left in the apparatus. From this it follows that our rectifications were, indeed, appreciably more effective than a simple evaporation, for which this ratio is about ½ (§ 2). On the other hand we did not yet reach conditions as efficient as we did with the neon rectification [3]).

We gladly record our thanks to Mr. H. KUIPERS, chief-glassblower, for his valuable help in the construction of the rectifying apparatus.

---

1) Professor ZEEMAN cooperating with Mr. DE GIER was so kind as to examine our sample VI with his mass spectrograph built on the principle of J. J. THOMSON's parabolae. He found a very distinct line corresponding to a mass 4, which line was not found in ordinary hydrogen, hence belongs to ions $H^1_2\ H^2$ (and H^2_2). He also found a line corresponding to mass 5, probably belonging to $H^1\ H^2_2$.

2) In putting this we make the following suppositions: 1. that in this calculation we may neglect the transformation ortho → para-hydrogen, and 2. that the evaporation process behaves in an ideal manner. A justification can be taken from the results of MEISSNER and STEINER's experiments. Their reduction in volume being about 4750 : 1, the resulting hydrogen according to the formula in § 2 could have contained 4.3 ‰ of H^1H^2 maximum. According to the measurement by mass spectrograph made by KALLMANN (cf. MEISSNER and STEINER's paper at the end) this concentration has really been attained.

3) Cf. these Proceedings **34**, 42, 1931. Comm. Leiden N⁰. 213*a* § 7.

Physics. — *Change of the resistance of metals in a magnetic field at low temperatures.* By W. J. DE HAAS and P. M. VAN ALPHEN. (Communication N⁰. 225*a* from the KAMERLINGH ONNES Laboratory, Leiden).

(Communicated at the meeting of March 25, 1933).

§ 1. *Introduction.* In previous papers we described the interesting results of our investigations on the change in resistance of single-crystals of very pure bismuth [1]) and on the simultaneous occurrence of a dependence of their diamagnetic susceptibility upon the magnetic field [2]). To discuss the question thoroughly however we didn't know enough about the change of the resistance in a magnetic field as a function of the temperature. Theoretically little can be said, and no wonder, for even the explication of the influence of the magnetic field on the resistance merely is entremely difficult. Especially, in trying to explain the order of magnitude of the longitudinal effect [3]) we meet with great difficulties.

We may mention here the detailed investigations of KAPITZA [4]), which furnish data for a very great number of materials. The main subject however of these investigations is not the change in resistance as a function of the temperature but rather as a function of the intensity of his very high magnetic field. The low temperatures (liquid air) were determined not very accurately and no measurements were made below 77° K.[5]).

For this reason we planned a series of measurements not only of the magnetic properties, but also of the change in resistance. As a matter of fact our attention was specially directed towards the region of hydrogen temperatures, though always the effect was also investigated at higher temperatures.

§ 2. *Measuring method.* The resistance measurements were made with the compensation apparatus of DIESSELHORST, as is usual in the

---

1) L. SCHUBNIKOW and W. J. DE HAAS, Proc. Amsterdam **33**, p. 130, 363, 1930. Comm. Leiden N⁰. 207.

2) W. J. DE HAAS and P. M. VAN ALPHEN. Proc. Amsterdam **33**, p. 680, 1106, 1930, **35**, p. 454, 1932. Comm. Leiden 208*d*, 212*a*, 220*d*.

3) R. GANS. Ann. der Phys. (4) 20, p. 293, 1906.
A. SOMMERFELD and N. H. FRANCK. Zeit. für Phys. **47**, p. 1, 1928; Review of Modern Physics 3, p. 1, 1931.
R. PEIERLS. Zeit. für Phys. **53**, p. 255, 1929

4) P. KAPITZA. Proc. Roy. Soc. **123**, p, 292, 1929.

5) For gold at 4,2° K. see W. MEISSNER and H. SCHEFFERS. Phys. Zeit. **30**, p. 827, 1929; Phys. Zeit. **31**, p. 574, 1930; Naturw. **18**, p. 110, 1929.

Leyden laboratory. The bifilar resistance wires were wound on a thin piece of mica, which could be turned over 90° about its longitudinal axis. The mica was placed alternately with its plane in the direction of the magnetic lines of force and perpendicular to these. In this way the longitudinal and the transverse effect are measured alternately. It is true, that in the first case the field and current are not exactly parallel through the screw of the windings. This deviation however is small and was afterwards proved to be proportional with $\cos^2 \alpha$. The advantage of this form of resistance is, that it enables us to make large resistances, which is very desirable for the measurements at low temperatures.

In order to prevent impurities by soldering, we fuse 4 platinum pieces to the resistances. The copper wires were then soldered at these pieces. Five of these resistances were mounted in the cryostat on a glass rod, which could be moved up and down from the outside by means of a packing tube. The angle of rotation was read on a divided circle in degrees.

As at low temperatures the resistance changes may become very large, the current of the magnet must be kept very constant. To this purpose we used the current of a large pile of accumulators.

The value of R/R_0 was an indication of the degree to which the wires had remained pure, for the purer the material, the smaller the value of R/R_0 at hydrogen temperatures. This control is of great importance, the possibility of the introduction of impurities during the fashioning being very great.

§ 3. *Alloys. Influence of impurities.* Discussing the conductivity of alloys, we must always bear in mind, that we may have to do with an impure metal. When e.g. the composition is such, that a metallic compound is formed with a lattice of its own, so that it must be regarded as a new metal, we must not forget that there will always be a small surplus of one of the components. This surplus forms an impurity in the new lattice. Therefore we never can simply compare an alloy with a pure metal.

Now it seems to be true that impurities have the same influence on the electric conductivity as a rise of temperature. Both give rise to an irregularity in the lattice and this increases the resistance.

Wishing to compare an impure metal with a pure material, we must do this at high temperatures.

Then the irregularities caused by the impurities are small compared with those due to the high temperature. When the temperature is lowered we come in a region, where the influence of the admixtures (which does not alter with the temperature) becomes great compared with that of the heat motion. Further cooling down has no influence, the resistance becomes independent of the temperature and its value depends on the quantity of the admixture. All this is duly represented by the rule of MATHIESSEN [1]).

---

[1]) H. MATTHIESEN, Pogg. Ann. **100**, p. 190, 1860; **122**, p. 19. 1869.

For the change of the resistance in a magnetic field a similar rule seems to hold, as will be evident from the next paragraph.

TABLE 1. (23.6 K G).

Temp.	Cu			Zn			Cu + Zn		
	R_{0T}/R_0	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	R_{0T}/R_0	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	R_{0T}/R_{00}	$\Delta R/R_{00}$	$\Delta R/R_{0T}$
290° K.	—	0.0000	0.000	—	0.00036	0.00036	—	0.0039	0.0039
77.2	0.148	17	0.009	0.202	0.0036	0.018	0.449	0.011	0.022
70.4	0.122	17	0.014	0.175	0.0043	0.024	0.433	0.011	0.027
63.8	0.094	17	0.021	0.143	0.0053	0.036	0.411	0.011	0.025
20.4	0.020	41	0.192	0.012	0.0172	1.377	0.298	0.011	0.042
17.3	0.020	41	0.198	0.009	0.0191	2.050	0.282	0.011	0.047
14.2	0.020	41	0.191	0.007	0.0218	3.103	0.280	0.011	0.051

§ 4. *Cu-Zn-Alloy.* As has been mentioned earlier [1]) this alloy has an abnormally high diamagnetism, which is much stronger than that of its components. We thought it interesting to investigate the change of resistance for this material more thoroughly. The results of the measurements are given in table I. At room temperature the change in resistance of an alloy of 30 % *Cu* and 70 % *Zn* is ten times that of pure zinc, though this too shows a rather great effect. No effect of the copper could be detected with our arrangement. Evidently the change in resistance is much stronger than in the case of the pure components. Moreover the susceptibility has strongly increased, so that we find in this alloy a new example of the simultaneousness of a high diamagnetism and a strong change of resistance, a phenomenon to which in 1914 already the attention was drawn [2]). This alloy belongs to the group of metals, for which the hypothesis of EHRENFEST [3]) probably holds viz. of the metals with a strong diamagnetism (bismuth, antimony, gallium and probably also graphite).

The agreement with this group lies in the fact that melting causes a steep fall of the strong diamagnetism to a much lower value. Moreover, in contrast with the case of other metals, the conductivity seems to increase by the melting.

Considering now the change with the temperature, we see that the impurities begin to play a role. For *Zn* and *Cu* the change increases always; for the alloy however the change of the resistance in the magnetic

[1]) K. GREULICH. Sitz. Ber. Rostock, p. 227, 1915.
K. OVERBECK. Ann. d. Phys. **46**, p. 677, 1915.
H. ENDO. Sc. Rep. Tohoku Un. **14**, p. 479, 1925; **16**, p. 201, 1927.
[2]) W. J. DE HAAS. Proc. Amsterdam **16**, p. 1110, 1914.
[3]) P. EHRENFEST. Physica **5**, p. 388. 1925; Zeit. für Phys. **58**, p. 719, 1929.

field depends no longer on the temperature below 77°.2 K. That is why at hydrogen temperatures the zinc shows a stronger effect than this alloy.

TABLE 2. Zinc.

H K Gauss	$T = 20°.4$ K.		$T = 14°.2$ K.	
	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	$\Delta R/R_{00}$	$\Delta R/R_{0T}$
1.0	0.0003	0.016	0.0002	0.040
3.1	14	0.100	14	0.216
5.2	26	0.205	29	0.422
7.8	43	0.343	50	0.704
10.3	62	0.488	69	1.007
14.6	96	0.753	112	1.617
20.1	141	1.122	177	2.521
23.5	0.0172	1.377	0.0220	3.146

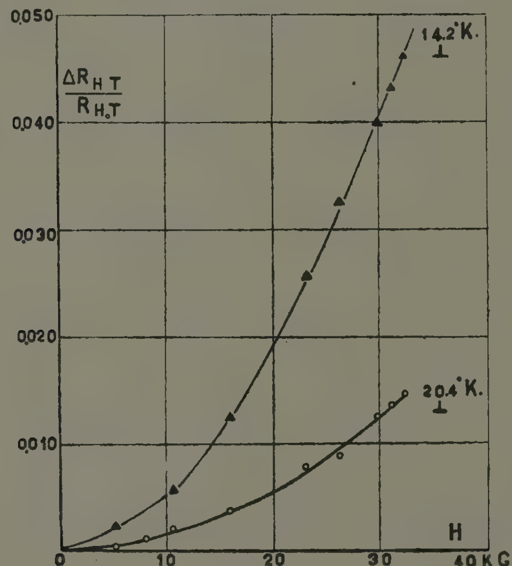


Fig. 1

§ 5. *Cd-Hg-Alloy*. Beside the pure components we investigated an alloy *Cd* with 1 % Hg and an alloy *Cd* with 30 atom % Hg. This latter alloy has the same crystal lattice as the pure cadmium, so that the mercury is dissolved in the cadmium. By long annealing, this resistance was made very homogeneous, while in the case of the 1 % alloy not much attention was paid to homogeneity.

From table III we see, that at room temperature no difference is seen

between the pure cadmium and the cadmium with 1 % admixture of mercury. At low temperatures however this difference becomes evident.

TABLE 3. (23.6 (K Gauss).

Temp.	Cd			Cd $1^0/_0$ Hg			Cd + $30^0/_0$ Hg		
	R_{0T}/R_{00}	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	R_{0T}/R_{00}	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	R_{0T}/R_{00}	$\Delta R/R_{00}$	$\Delta R/R_{0T}$
290°K.	—	0.0022	0.0020	—	0.0023	0.0022	—	—	—
77.2	0.2513	0.00954	0.0379	0.4298	0.0056	0.0132	0.406	0.003	0.005
63.8	0.1919	0.01027	0.0533	0.3844	0.0064	0.0165	0.363	0.003	0.008
20.4	0.02297	0.02672	1.163	0.2627	0.0144	0.0549	0.232	0.003	0.018
14.2	0.00811	0.03345	4.132	0.2459	0.0157	0.0638	0.215	0.003	0.019

Interesting is also the difference in the course of $\Delta R/R_{H_0 0}$ and $\Delta R/R_{H_0 T}$ where $R_{H_0 0°}$ and $R_{H_0 T}$ are the values of the resistance without magnetic field at 0° C and at the temperature T for which ΔR was determined respectively.

For the cadmium with 30 % Hg $\Delta R/R_{H_0 0°}$ is seen to become independent of the temperature. Here the influence of the admixture predominates by far over that of the temperature. The change in resistance is much smaller. In table IV and fig. II the resistance has been given as a function of the field and as a function of the temperature.

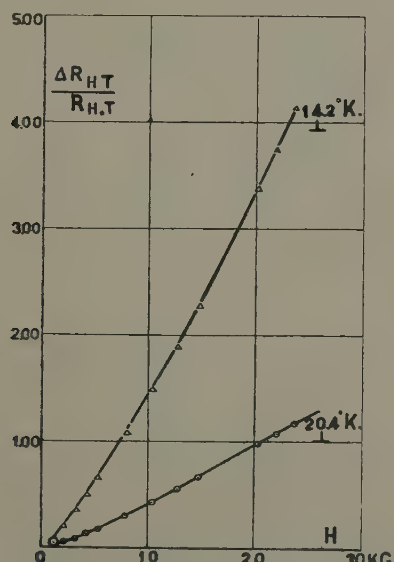


Fig. 2

TABLE 4. Cadmium.

H	$T = 290°$ K.		$T = 77°.2$		$T = 63°.8$		$T = 20°.4$		$T = 14°.2$	
	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	$\Delta R/R_{00}$	$\Delta R/R_{0T}$
1.0	—	—	—	—	0.00003	0.00015	0.00028	0.012	0.00028	0.036
2.1	—	—	0.00010	0.00040	15	78	96	42	157	196
3.1	—	—	30	115	33	173	183	80	274	340
4.1	—	—	47	184	61	320	291	122	400	495
5.2	0.00007	0.00006	74	298	93	483	389	170	526	651
7.8	—	—	154	614	176	917	686	299	863	1067
10.3	40	37	247	981	284	1478	986	430	1196	1478
12.6	—	—	351	1396	414	2150	1278	557	1524	1883
14.6	146	135	449	1783	513	2663	1522	663	1820	2249
20.1	166	154	749	2974	825	4281	2236	974	2725	3367
21.7	—	—	839	3333	923	4788	2447	1.065	3023	3.734
23.5	0.00219	0.00203	0.00954	0.03787	0.01038	0.05325	0.02672	1.163	0.03345	4.132

§ 6. *Hg with 50 atom % Cd.* This alloy crystallizes in the Hg lattice.

TABLE 5. (23.6 K Gauss).

Temp.	Hg			50 % Hg + 50 % Cd		
	R_{0T}/R_{00}	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	R_{0T}/R_{00}	$\Delta R/R_{00}$	$\Delta R/R_{0T}$
77.2 °K	0.06626	—	—	0.305	—	—
63.8	0.05317	—	—	0.290	—	—
20.4	0.01465	0.000109	0.0074	0.253	0.0018	0.0007
14.2	0.00817	0.000211	0.0268	0.247	0.0018	0.0007

As to the Hg we must bear in mind, that it is measured in a glass tube. At low temperatures it becomes solid and we don't know, whether it is then tension free and polycrystalline. For $R_{H_0 0}$ the value for the liquid Hg has been taken.

§ 7. *Solid solution. Crystal of Ti and Zr*[1]. The change in resistance was rather small and the rods were rather thick, so that the resistance change could not be measured very accurately. So not many conclusions can be drawn. The change is smaller than in the case of *Zr*. The composition of the alloy was not known.

[1]) These rods were kindly put at our disposal by Dr. G. DE BOER of the Philips Laboratory, Eindhoven.

TABLE 6. Mercury.

H K. Gauss	T = 20°.4 K. $\Delta R/R_{0T}$	T = 14°.2 K. $\Delta R/R_{0T}$
5.2	0.0004	0.0023
8.1	11	—
10.6	20	57
15.9	36	125
23.1	78	256
26.2	86	316
29.8	124	398
32.2	0.0135	430
32.3	0.0145	0.0459

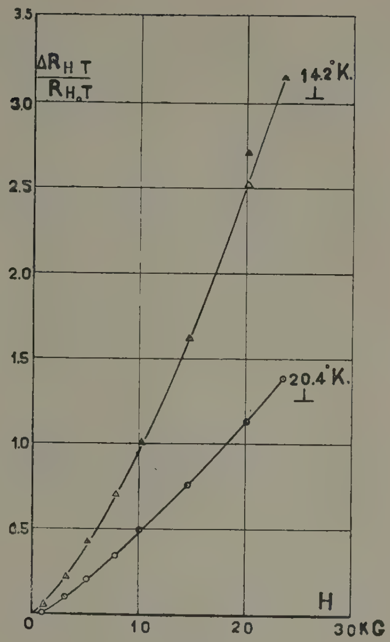


Fig. 3

TABLE 7. (23.6 K. Gauss).

Temp.	Ti R_{0T}/R_{00}	Ti $\Delta R/R_{00}$	Ti $\Delta R/R_{0T}$	Zr R_{0T}/R_{00}	Zr $\Delta R/R_{00}$	Zr $\Delta R/R_{0T}$	Ti + Zr R_{0T}/R_{00}	Ti + Zr $\Delta R/R_{00}$	Ti + Zr $\Delta R/R_{0T}$
77°.2 K.	0.2724	—	—	0.2087	0.0005	0.0025	0.369	—	—
63.8	0.2445	—	—	0.1566	0.0007	0.0047	—	—	—
20.4	0.1423	0.0002	0.002	0.0594	0.0020	0.0330	0.222	0.0005	0.0022
14.2	0.1394	0.0002	0.002	0.0568	0.0021	0.0370	0.219	0.0004	0.0018

TABLE 8. Zircoon.

H	T = 14°.2 K $\Delta R/R_{00}$	T = 14°.2 K $\Delta R/R_{0T}$
3.1	0.00006	0.0011
5.2	13	24
10.3	36	62
14.6	92	159
20.1	154	272
21.7	173	304
23.5	0.00206	0.0360

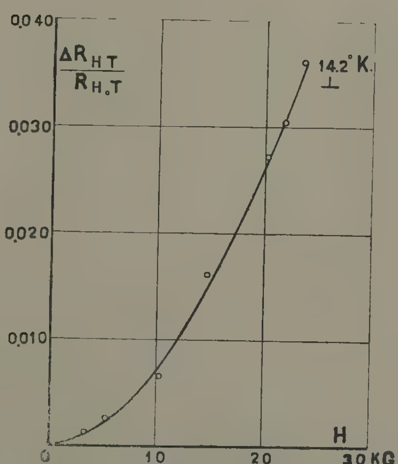


Fig. 4.

§ 8. *Transition from the transverse to the longitudinal effect.* The high values of the resistance change at low temperatures enabled us to measure the effect very accurately. In two cases we even investigated precisely how the resistance change depended upon the angle α (between the direction of the measuring current and that of the magnetic field). The two extreme cases are $\alpha = 0°$ and $\alpha = 90°$ giving resp. the longitudinal and the transverse effect.

We investigated a pure metal (*Al*) and an impure metal (*Cd* with 1 % Hg). Both cases showed a similar behaviour. The resistance change for an angle α is represented by the formula:

$$\triangle R_\alpha = \triangle R_{//} \cos^2 \alpha + \triangle R_\perp \sin^2 \alpha,$$

where $\triangle R_{//}$ and $\triangle R_\perp$ are the values of $\triangle R$ in the longitudinal and the transverse case respectively. This expression is the same as that found both theoretically [1]) and experimentally [2]) for the conductivity of a mono-axial crystal at room temperature in a direction making an angle α with the principal axis.

In the originally isotropic metal a direction is fixed by the magnetic field. In directions parallel and perpendicular to this direction the metal has different conductivities. The resistance in an intermediate direction is found in the same way as for a mono-axial crystal.

TABLE 9. (23.6 K Gauss) $\triangle R/R_{00}$

Cd 1 % Hg.

α	Measured	Calculated	Difference
15°	0.00966	0.00966	0.00000
0	932	932	0
15	959	966	7
25	1015	1022	7
35	1092	1100	8
45	1181	1166	5
55	1270	1274	4
65	1354	1350	4
75	1410	1406	4
90	1441	1441	0
75	1409	1406	3
65	0.01348	0.01350	2

AL.

α	Measured	Calculated	Difference
17°	0.00231	0.00223	0.00008
7	207	207	0
2	202	202	0
3	202	202	0
13	217	215	2
23	246	238	8
33	285	272	13
43	321	315	6
53	351	350	1
63	391	387	4
73	415	414	1
83	429	430	1
87	432	432	0
77	0.00422	0.00421	1

The deviations between the calculated and measured values are greatest

1) W. VOIGT. Kristallphysik. Teubner 1910, section 173.

2) P. BRIDGEMAN. Proc. Amer. Acad. **60**, p. 305, 1924.
E. GRUNEISEN and E. GOENS. Zeit. für Phys. **26**, p. 250, 1924.

in the middle, because here an inaccuracy in the adjusting of the angle α has the greatest influence. As the reading of the angle was accurate to

TABLE 10. *Al* Transversal.

H K. Gauss	$T=77°.2$ K.		$T=20°.4$ K.		$T=14°.2$ K.	
	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	$\Delta R/R_{00}$	$\Delta R/R_{0T}$
1.0	—	—	0.00002	0.0002	0.00001	0.0002
2.1	0.00003	0.0002	6	8	7	11
3.1	9	6	—	—	13	21
4.1	9	6	23	33	24	38
5.2	12	8	—	—	36	56
7.8	22	14	71	103	75	114
10.3	40	25	117	172	123	185
12.6	61	39	—	—	171	258
14.6	77	48	208	308	217	326
20.1	139	87	344	509	358	539
21.7	157	98	387	574	403	605
23.5	0.00177	0.0110	0.00432	0.0640	0.00451	0.0678

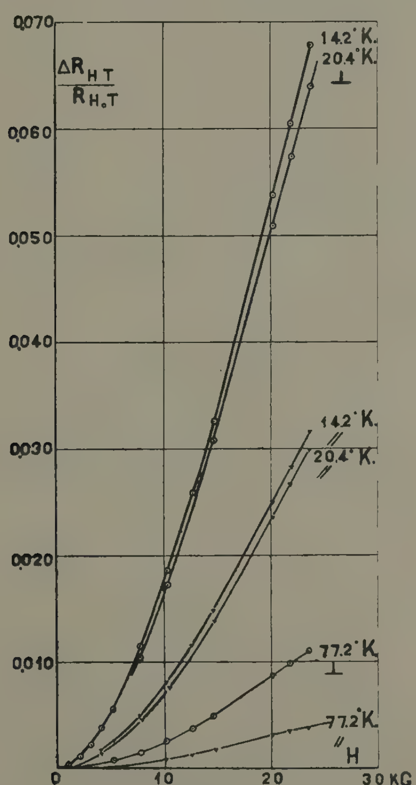


Fig. 5

1°, the agreement between the calculated and measured values may be called satisfactory. The behaviour of *Al* in different fields may be found in table 10.

TABLE 11. Al. Longitudinal.

H K Gauss	$T = 77°.2$ K.		$T = 20°.4$ K.		$T = 14°.2$ K.	
	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	$\Delta R/R_{00}$	$\Delta R/R_{0T}$	$\Delta R/R_{00}$	$\Delta R/R_{0T}$
1.0	—	—	0.00001	0.0001	—	—
2.1	—	—	2	4	0.00002	0.0003
3.1	—	—	—	—	6	9
4.1	—	—	9	13	10	16
5.2	0.00004	0.0003	—	—	16	24
7.8	4	3	31	46	32	49
10.3	12	7	51	75	54	80
12.6	20	12	—	—	77	114
14.6	28	17	93	138	98	146
20.1	53	33	160	236	166	248
21.7	56	35	180	266	188	281
23.5	0.00060	0.0038	0.00202	0.0299	0.00211	0.0316

§ 9. *Summary.*

1. A high diamagnetism and a strong resistance change go together.
2. Impurities have a greater influence upon the resistance change at low temperatures than at high temperatures.
3. Alteration of the angle between the directions of the current and of the field viz. transition from the transversal case to the longitudinal one, changes the resistance in the same way as the alteration of the angle between principal and secondary axes does for a mono-axial crystal.

For crystals the phenomena are much more complicated.

Finally we wish to express our thanks to Mr. J. DE BOER for kindly preparing the *Cd-Hg*-alloys and to Mr. O. GUINAU for his valuable help in the measurements and the calculations of the results.

Physics. — *Magnetic properties of metals at low temperatures.* By W. J. DE HAAS and P. M. VAN ALPHEN. (Communication N⁰. 225*b* from the KAMERLINGH ONNES Laboratory Leyden.)

(Communicated at the meeting of March 25, 1933).

§ 1. *Introduction.* The extensive investigations of K. HONDA [1]) and M. OWEN [2]) show, that the law of CURIE [3]) on the independence of the diamagnetic susceptibility of the temperature is experimentally stated in very few cases only. For 21 of the 28 elements examined the susceptibility changes with the temperature between —170° and 1200°, generally it becomes weaker when the temperature is raised.

Theoretically it may seem allowed to calculate the diamagnetism of free ions and atoms with the aid of the LARMOR theorem [4]). For solid bodies this is however not allowed the cristal fields giving rise to a non-symmetrical field of force.

The experiments show that for solid bodies the outward electron has a very great influence upon the diamagnetism. The field in which the electron is moving will evidently change with the heat motion, so that an influence of the temperature on the diamagnetism might be expected.

In so far as the conduction electrons in a metal may be regarded as free the difference between the quantum theory and the classical theory [5]) is that the former ascribes to the spin a weak paramagnetism [6]), independent of the temperature and neutralized for one third by a diamagnetic effect of the orbital motion [7]).

Experiments on the influence of impurities [8]) and tensions [9]) on the

---

1) K. HONDA. Ann. der Phys. **32**, 1027, 1910.
2) M. OWEN. Ann. der Phys. **37**, 657, 1912.
3) P. CURIE. Ann. Chim. et Phys. **7**, 289, 1895.
4) J. LARMOR. Phil. Mag. (5), **44**, 503, 1897.
5) H. A. LORENTZ. Wolfskehl Vorträge. 1913.
6) W. PAULI. Zs. f. Phys. **41**, 81, 1927.
J. FRENKEL. Zs. f. Phys. **49**, 31, 1928.
7) L. LANDAU. Zs. f. Phys. **64**, 629, 1930.
BITTER. Nat. Ac. of Sc. **16**, 95, 1930.
C. G. DARWIN. Proc. Cambr. Phil. Soc. **27**, 86, 1931.
E. TELLER. Zs. f. Phys. **67**, 311, 1931.
8) A. GOETZ and A. B. FOCKE. Phys. Rev. **38**, 1569, 1931.
9) SEEMANN and E. VOGT. Ann. der Phys. (5) **2**, 976, 1929.
BITTER. Phys. Rev. **36**, 498, 1930.
K. HONDA and Y. SHIMIZU. Nature **126**, 990, 1930.
F. W. LOWANCE and F. W. CONSTANT. Phys. Rev. **38**, 1547, 1931.

magnetic susceptibility show very clearly that the magnetic susceptibility like the conduction of electricity has its seat at the exterion of the atom.

As a rule all factors that influence on electrical conductivity do so with the diamagnetism, though to a lower degree.

For this reason we thought it interesting to investigate beside the changes in resistance at low temperatures also the magnetic properties of the same substances.

§ 2. *Apparatus.*

The apparatus used was the same as that described in our papers on the susceptibility of single-crystals of bismuth [1]). We worked with the rod method and with eight different field intensities (maximum 24 K Gauss).

When the susceptibility increases with the field, a correction can be added for admixtures of iron with the aid of the formula: $\chi = \chi_{H\infty} - \frac{A}{H}$.

The cases, in which this proved necessary have been marked.

For the most part the rods were cast in glass or quartz in vacuum. The diameter was ± 5 mm and the length 10 cm. The surface was thoroughly etched with an acid to remove impurities. The specific weight was determined for the calculation of the specific susceptibility χ. The volume susceptibility was not corrected for the contraction at low temperatures; it was calculated from the dimensions of the rod as room temperature.

§ 3. *Results.*

Graphite. This was a pressed specimen of powdered Ceylon graphite (DE HAËN) purified with H Fl. By the easy shear in a direction perpendicular to the principal axis the pressing has a directing influence upon the particles, so that the result depends rather too much on the susceptibility along the binary axes.

Graphite.

T	$\chi \times 10^6$
289° K.	— 3.0
77.2	— 4.7
63.8	— 4.7
20.4	— 4.7
14.2	— 4.7

Copper.

T	$\chi \times 10^6$	$\varkappa \times 10^6$
289° K.	— 0.086	— 0.746
77.2	0.090	0.781
63.8	0.091	0.787
20.4	0.097	0.84
14.2	0.097	0.84

[1]) W. J. DE HAAS and P. M. VAN ALPHEN. Proc. Acad. Amsterdam **33**, 680, 1930; **33**, 1106, 1930;

A correction has been applied for small admixtures of iron. We did not find here any periodical dependence of the susceptibility upon the intensity of the field as we did for single-crystals of bismuth at low temperatures.

Different samples of crystallized graphite gave always a constant susceptibility at low temperatures. By the impurities however they had always been rendered paramagnetic at low temperatures.

The susceptibility of graphite increases with 57 %.

Copper. This was furnished by the Bureau of Standards Washington. The susceptibility depended upon the field-intensity, so that probably no ferromagnetic admixture is present. Beforehand the rod had been well annealed to eliminate the influence of tensions. By the good conductivity the measurements at low temperatures are not so accurate through the great damping.

The susceptibility increases with 12 %.

Zinc (quality KAHLBAUM). The change at low temperature suggests a weakly paramagnetic impurity.

Alloy 30 % Cu and 70 % Sn. From the above copper and zinc we prepared this alloy, which is strongly diamagnetic [1]. The susceptibility is much greater than that of the components and changes little with the temperature. Probably the compound contains paramagnetic impurities.

Zinc

T	$\chi \times 10^6$	$\varkappa \times 10^6$
289° K.	— 0.143	— 0.987
77.2	0.175	1.210
63.8	0.172	1.185
20.4	0.170	1.17
14.2	0.170	1.17

Alloy Cu—Sn.

T	$\chi \times 10^6$	$\varkappa \times 10_9$
289° K.	— 0.79	— 5.92
20.4	0.78	5.88
14.2	0.77	5.79

It is remarkable that this alloy, which for the greater part consists of the χ phase [2] shows a much greater resistance change in a magnetic field than zinc or copper. This is a new confirmation of the connexion found already earlier by us between magnetic properties and resistance change [3].

[1] K. GREULICH. Sitz. Ber. Rostock 227, 1915.
K. OVERBECK. Ann. der Phys. **46**, 677, 1915.
H. ENDO. Sc. Rep. Tohoku Un. **14**, 479, 1925; **16**, 201, 1927.

[2] A. WESTGREN and G. PHRAGMEN. Phil. Mag. **50**, 311, 1925.

[3] W. J. DE HAAS. Proc. Amsterdam **16**, 1110, 1914.

Silver. (H. S. BRAND-HILGER). Here too the measurements at low temperatures are not so accurate through the great damping.

Cadmium (KAHLBAUM). The diamagnetic susceptibility of cadmium strongly increases (vir. with 78 %), when the temperature is lowered. This is the greatest change yet observed by us. Moreover, it is remarkable,

Silver.

T	$\chi \times 10^6$	$\varkappa \times 10^6$
289° K.	— 0.188	— 1.89
77.2	0.191	192
63.8	0.190	191
20.4	0.19	1.9
14.2	0.19	1.9

Cadmium.

T	$\chi \times 10^6$	$\varkappa \times 10^6$
289° K.	— 0.183	— 1.53
77.2	0.245	2.05
70.4	0.250	2.10
63.8	0.256	2.15
20.4	0.319	2.67
14.2	0.326	2.73

that the susceptibility still increases in the region of the liquid hydrogen temperatures, which occurs very seldom. In by far the most cases in this temperature region the susceptibility is independent of the temperature.

The susceptibility is independent of the field-intensity.

Tin. This metal was used in two modifications viz. as the paramagnetic white tin (KAHLBAUM) and as the diamagnetic grey tin. The latter was prepared by Prof. KRUYT and Prof. COHEN in the VAN 'T HOFF laboratory and was used for the research on the conductivity. The grain dimensions were 0.1 to 0.5 mm. The sample contained a ferromagnetic impurity and probably also a paramagnetic one.

Tin (white).

T	χ 10^6	K 10^6
289° K.	+ 0.026	+ 0.187
20.4	0.024	0.173

Tin (grey).

T	$\chi \times 10^6$
289° K.	— 0.25
77.2	0.22
63.8	0.21[5]
20.4	0.20
14.2	0.18

Yet we may conclude from our investigations, that the prediction of BLOCH[1]) is not fulfilled.

From his theory of conduction of electricity combined with measurements of the specific heat BLOCK draws namely the conclusion, that below 70° K grey tin will become more strongly paramagnetic than the alkali-metals. We found however, that at 14°,2 it is still diamagnetic so that probably another interpretation must be given to the course of the specific heat.

Finally we may remark, that white tin becomes supraconductive and grey tin not[2]).

Antimony. We used a small single-crystalline piece, cleaved from a large block in a direction perpendicular to the principal axis.

No phenomena were observed similar to those found with single-crystals of bismuth. The sample contained however a weakly ferromagnetic impurity and probably was not so well crystallised as the single-crystals of bismuth, so that the measurements will have to be repeated later on with better crystals.

Tellurium (KAHLBAUM). As will be known tellurium shows dynamical allotropy; its properties strongly depend on the previous history of the sample. We melted the tellurium furnished by KAHLBAUM in the form of rods. Then a rod of 5 mm diameter and 10 cm length was cast and quickly tempered.

Antimony.

T	$\chi \times 10^6$
289° K.	— 0.66
20.4	0.65
14.2	0.64

Tellurium.

T	$\chi \times 10^6$	$K \times 10^6$
289° K.	— 0.290	— 1.75
20.4	0.291	1.76
14.2	0.293	1.77

The susceptibility is found still to increase when the temperature is lowered, much less however than has been found by OWEN.

Therefore a more detailed investigation on the influence of a shift of the equilibrium on the susceptibility is very desirable.

The electric conductivity too is strongly altered by the melting process.

[1]) F. BLOCH. l. c.

[2]) W. J. DE HAAS, G. J. SIZOO and J. VOOGD. Versl. Amsterdam 36, 1227, 1927. Comm. Leiden 187*d*.

Iodium. The iodium was purified by means of sublimation. The susceptibility decreases with the temperature and shows still a weak change in the region of liquid hydrogen. This suggests the occurrence of a paramagnetic impurity.

Iodium.

T	$\chi \times 10^6$
289° K.	— 0.336
20.4	0.316
14.2	0.313

Wolfram.

T	$\chi \times 10^6$	K 10^6
289° K.	+ 0.284	+ 5.60
20.4	0.287	5.67
14.2	0.292	5.76

Wolfram (H. S. BRAND HILGER). The very pure wolfram (no ferromagnetic impurities) is paramagnetic. The susceptibility increases weakly at low temperatures.

Gold (H. S. BRAND HILGER). The susceptibility weakly decreases, probably by a little admixture of iron, for which we applied a correction.

Gold.

T	$\chi \times 10^6$	$K \times 10^6$
289° K.	— 0.139	— 2.63
70.4	0.134	2.54
14.2	0.132	2.50

Lead.

T	$\chi \times 10^6$	$K \times 10^6$
289° K.	— 0.111	— 1.25
77.2	0.129	1.45
63.8	0.131	1.47
20.4	0.132	1.48
14.2	0.132	1.48

Lead (KAHLBAUM). The increase amounted to 35 %. No ferromagnetic admixtures occurred.

Thallium (KAHLBAUM). The thallium was very pure without a trace of an admixture of iron. When cooled down its susceptibility increases with 20 %.

Magnesium (H. S. BRAND HILGER). As at high temperatures magnesium forms a compound with glass and quartz, the samples were prepared in iron vessels. The magnesium used by us was according to HILGER, "of the highest possible degree of purity". Yet it contains still 0.047 % *Fe*. As might have been expected from this admixture, the susceptibility

depended rather strongly upon the field, for which a correction was added.

The gradual change of the susceptibility at the lowest temperatures

Bismuth.

T	$\chi \perp$ head-axis	χ//head-axis
289° K.	— 1.48	— 1.045
77.2	— 1.80	—
63.8	— 1.80	—
20.4	field dependant	— 1.20
14.2	field dependant	— 1.20

Thallium.

T	$\chi \times 10^6$	$K \times 10^6$
289° K.	— 0.215	2.47
77.2	0.246	2.83
63.8	0.248	2.84[5]
20.4	0.255	2.92[5]
14.2	0.258	2.96[5]

suggests, that the influence of the iron on the course of the susceptibility-temperature curve is not so great as might be expected at first.

Copper—Tin.

T	$\chi \times 10^6$	$K \times 10^6$
289° K.	— 0.323	— 2.86
71.2	0.332	2.94
20.4	0.332	2.94
14.2	0.330	2.92

Magnesium.

T	$\chi \times 10^6$	$K \times 10^6$
289° K.	+ 0.94	+ 1.61
77.2	0.98	1.68
20.4	1.13	1.93
14.2	1.13	1.93

Niobium (H. S. BRAND HILGER). According to the statement this metal contained traces of tin and iron. Of the latter we detected no influence at all. Perhaps it occurs in a combinasion?

The susceptibility increases with 2 %.

Niobium.

T	$\chi \times 10^6$	$K \times 10^6$
289° K.	+ 2.28	+ 19.2
20.4	2.34	19.7
14.2	2.34	19.7

Molybdenum.

T	$\chi \times 10^6$	$K \times 10^6$
289° K.	+ 0.949	+ 9.54
77.2	1.080	10.85
63.8	1.127	11.33
20.4	1.555	15.62
17.3	1.664	16.72
14.2	1.819	18.28

Molybdenum (H. S. BRAND HILGER). Spectroscopically some elements occurred as impurities; chemically they could not be detected. Iron did not occur. The paramagnetic susceptibility increases with 100 % and is strongly dependent on the temperature, even at the temperatures of liquid hydrogen.

Bismuth (SCHUBNIKOW)[1]. Finally we give the results of the measurements with single-crystals of very pure bismuth, which have been published already[2].

Alloy of Cu and Sn. This rod was procured us by Prof. VAN AUBEL. According to his statement it was a sample of the compound Cu_4Sn. The susceptibility of this compound is again much greater than that of the components and increases still a little, when cooled down.

§ 4. *Summary.*

The course of the susceptibility in the region of liquid hydrogen temperatures (20°.4 K. to 14°.2 K.) gives us a good indication about the purities of the samples.

Generally this temperature interval will be too small for changes in the diamagnetic susceptibility, the more so because (as is shown by experiment) it does not strongly depend upon the temperature. This cannot be said of the paramagnetic susceptibility. This shows a strong change with the temperature, especially at low temperatures. When a diamagnetic substance contains such an impurity, its susceptibility will decrease at low temperatures.

Our research shows, that some metals (zinc and cadmium) for which OWEN found at low temperatures a susceptibility independent of the temperature, become a little stronger diamagnetic and that the susceptibility of silver does not decrease but increases.

The result is that of the 21 diamagnetic metals only 4 (*Si, Se, Cs, Au*) have a susceptibility, which remains, constant throughout the whole range of temperature.

According to OWEN the susceptibilities of borium, beryllium and mercury decrease with the temperature. These results may however be due to small impurities.

Most metals show the opposite behaviour. The susceptibility is first constant and decreases then numerically when the temperature is raised.

It would be interesting to find an explication of these facts with the aid of the modern views on the behaviour of the electrons in a metal.

Finally we wish to express our thanks to Mr. GUINEAU for his valuable help during the measurements.

---

[1] L. SCHUBNIKOW and W. J. DE HAAS. Proc. Amsterdam 33, 350, 1930. Comm. Leiden 207c.

[2] W. J. DE HAAS and P. M. VAN ALPHEN. l. c.

Physics. — *Sparks in air of atmospheric pressure.* By G. HOLST.

(Communicated at the meeting of March 25, 1933).

R. STRIGELS's experiments [1]) show that at voltages as high as 180 % of the static sparking potential, every electron leaving the cathode, produces a spark. The discharge takes place between spheres of 5 cm diameter, at a distance of 1.1 mm, the static sparking potential being 5000 Volts. A large condenser, charged by a transformer and rectifier, supplies the current.

F. G. DUNNINGTON [2]) studied the early stages of breakdown by his electro-optical shutter and found that a luminous filament is formed in the space between the electrodes, which starts from the cathode and may or may not proceed to the anode. This shows that the breakdown takes place in a very limited part of space.

The time between the moment the voltage on the gap rises above the sparking potential and the moment this voltage drops to a small value again, may be divided in two periods: *a* the time before an electron leaves the cathode, *b* the time during which the ionisation in the gas increases to such an extent that the condenser discharges.

This latter period will be discussed hereafter.

W. ROGOWSKI [3]) has demonstrated that this time may be extremely short, 10^{-7} or even 10^{-8} sec. He pointed out, that these short times cannot be accounted for by TOWNSEND's theory of the sparking potential. The potential difference per free path being very small, it is probable that + ions will be able to liberate fresh electrons only at the cathode surface. The time, during which the discharge develops, is thus determined by the speed of the + ions and this is not by any means sufficient to account for such short times 10^{-7} or 10^{-8} sec.

A. v. HIPPEL and J. FRANCK [4]) suppose that space charges can give the explanation. The electrons formed by ionisation move quikly to the anode, leaving the slow + ions in the gap, resulting in a concentration of the field near the cathode. This increases the ionizing power of the following electrons.

At lower gas-pressures the discharge takes place a great deal more slowly and can be explained by TOWNSEND's theory [5]). H. VIEHMANN [6])

1) R. STRIGEL. Wissensch. Veröff. Siemens Konzern (11), 52, 1932.
2) F. G. DUNNINGTON. Phys. Rev. (38), 1535, 1931.
3) W. ROGOWSKI. Arch. f. Electr. (16) 496, 1926 and following àrticles.
4) A. v. HIPPEL and J. FRANCK. Zeitschr. f. Phys. (57), 696, 1929.
5) M. STEENBECK. Wissensch. Veröff. Siemens Konzern (9), 42, 1930.
6) H. VIEHMANN. Arch. Electr. (25), 253, 1931.

using ROGOWSKI's methods, found that the discharge time decreases with the rate, at which the voltage impressed on the gap rises and with the overvoltage. In the case that no overvoltage is applied, the discharge time is 10^{-4}—10^{-3} sec. in accordance with TOWNSEND's theory.

We now return to STRIGEL's experiments. At 180 % overvoltage every electron leaving the cathode produces a spark. At 0, 20, 40, 60 % overvoltage the number of pairs of ions generated by a single electron from the cathode is $2{,}2.10^4$, $2{,}7.10^9$, 5.10^{16}, 1.10^{24} resp.

Compare this to the number of electrons on the charged condenser, which is of the order of 0,01—0,1 μ F. Take 0,1 μ F. The impressed voltage being of the order of 10000 V, the charge of the — plate is equal to $0{,}1.10^{-6} \,.\, 10000 \,.\, 6{,}3.10^{18} = 6{,}3.10^{15}$ electrons. At 40 % overvoltage and more one electron leaving the cathode produces a number of ions sufficient to completely discharge the condenser, *without the need of a second electron liberated at the cathode.* Such discharges will take place very quickly.

It is easy to see that as long as $e^{\alpha l}$ in TOWNSEND's formula is small compared to the density of surface charge on the electrodes, + space charges will not materially help to produce the spark. The + ions will have to proceed to the cathode to do their duty.

If $e^{\alpha l}$ becomes of the order of these surface charges the field is distorted by + space charges, concentrating it near the cathode, thus enhancing the ionizing power of the following electrons. The + ions in the space add to build up the discharge current a long time before they reach the cathode.

At still higher values of e^{α} one electron leaving the cathode may generate a sufficient number of pairs of ions to completely discharge the condenser. The + ions produced will be contained in a channel of small cross section, which moreover contains a great number of excited molecules. These will return to their normal state only after some lapse of time. Any fresh electron entering this channel during the life of these excited molecules will find very favourable ionizing conditions. The same holds for + ions, moving through this channel to the cathode. This explains the highly conductive filament between the electrodes. It also shows why a spark spectrum is produced under conditions, where the potential difference per free path is exceedingly small.

Physics. — *Das Entstehen einer kanonischen Gesamtheit.* Von L. S. ORNSTEIN und W. R. VAN WIJK.

(Communicated at the meeting of March 25, 1933).

In seinem berühmten Buch "Principles in Statistical Mechanics" hat J. W. GIBBS ein aus vielen Teilchen bestehendes mechanisches System betrachtet, das mit einem zweiten System von sehr grossem Energie-

inhalt in Wechselwirkung steht. Er hat gezeigt dass in dem stationären Zustand die Energie des ersten Systems um einen gewissen Mittelwert herumschwankt und zwar in solcher Weise dass das System denselben Gesetzen folgt wie die von ihm definierte kanonische Gesamtheit. Auf welche Art eine willkürliche Gesamtheit bei der Annäherung an den stationären Zustand in die Kanonische übergeht, darüber hat er sich aber nicht ausgesprochen.

Wir möchten im Folgenden über einige quantitative Betrachtungen, welche wir zu diesem Problem angestellt haben, Mitteilung machen.

Dazu denken wir uns wiederum die beiden, von GIBBS betrachteten, mechanischen Systeme mit sehr grosser Zahl der Freiheitsgraden, in Wechselwirkung. Die Energie des ersten Systems sei klein im Vergleich zu derjenigen des zweiten Systems. Nun denken wir uns nach der GIBBSschen Art das erste System M-fach wiederholt in dem mehrdimensionalen Phasenraum abgebildet, wobei M eine sehr grosse Zahl ist, und fragen nach der Wahrscheinlichkeit dafür dass $N(\varepsilon)$ derartiger Systeme eine Energie zwischen ε und $\varepsilon + d\varepsilon$ haben.

Da wir Energie-austausch zwischen dem ersten und dem zweiten System zugelassen haben wird sich die Zahl der Systeme in einem gewissen Interval $\varepsilon \,.\,.\, \varepsilon + d\varepsilon$ mit der Zeit ändern; es ist also $N(\varepsilon)$ eine Funktion der Zeit. Wir möchten jetzt versuchen eine Gleichung für $N(\varepsilon, t)$ aufzustellen.

Um zu dieser Gleichung zu gelangen müssen wir uns überlegen wie sich die Energie der Systeme ändert während einer kurzen Zeit τ. Es liegt nahe anzunehmen dass die Energie-änderung aus einem Systematischen und einem von dem Zufall beherrschten Teil besteht.

Für den systematischen Teil liegt es nahe anzunehmen dass er proportional zur Entfernung der Energie von einer karakteristischen Energiewert ε_0, (die mittlere Energie im stationären Zustande!) ist. Wir setzen für diesen Teil

$$-\beta(\varepsilon - \varepsilon_0)\,\tau$$

wo β eine Proportionalitätskonstante ist.

Weiter gibt es ein zufälliger Teil $\triangle$ der Energie-Aenderung wovon wir annehmen dass sein Mittelwert $\overline{\triangle} = 0$ und $\overline{\triangle^2} = D\tau$ ist; die Wahrscheinlichkeit für einen Wert von $\triangle$ zwischen $\triangle$ und $\triangle + d\triangle$ schreiben wir $W(\triangle)\,d\triangle$.

Es besteht also zwischen ε, dem Wert der Energie zur Zeit $t + \tau$, und ε', demjenigen zur Zeit t, die Beziehung

$$\varepsilon = \varepsilon' - \beta t(\varepsilon' - \varepsilon_0) + \triangle. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Wenn wir jetzt zur Zeit τ eine gewisse Verteilung $N(\varepsilon, \tau)\,d\varepsilon$ haben, so kann man leicht die Verteilung zur Zeit $t + \tau$ durch eine Differentialgleichung ausdrücken.

18

Denn es ist:

$$N(\varepsilon, t+\tau)\, d\varepsilon = \int N(\varepsilon', t)\, W(\triangle)\, d\triangle\, d\varepsilon'$$

wobei

$$\varepsilon = \varepsilon' - \beta\tau(\varepsilon' - \varepsilon_0) + \triangle.$$

Daher ist

$$d\varepsilon = d\varepsilon'(1-\beta\tau).$$

Also

$$N(\varepsilon, t+\tau)(1-\beta\tau) = \int N(\varepsilon + \beta\tau(\varepsilon-\varepsilon_0) - \triangle, t)\, W(\triangle)\, d\triangle.$$

Wo $\varepsilon - \varepsilon_0$ mit τ multipliziert wird kann man ε' durch ε ersetzen. Entwickelt man und geht man bis zur ersten Ordnung in τ so erhält man

$$\frac{\partial N(\varepsilon, t)}{\partial t} = \frac{\partial}{\partial \varepsilon}\{\beta(\varepsilon-\varepsilon_0) N(\varepsilon, t)\} + \frac{D}{2}\frac{\partial^2 N(\varepsilon, t)}{\partial \varepsilon^2} \quad . \; . \; . \quad (2)$$

Diese Gleichung ist die generalisierte Diffusionsgleichung die auch in der Theorie der BROWNschen Bewegung eine grosse Rolle spielt.

Untersuchen wir nun erst die physikalische Bedeutung der stationären Lösung $\left(\frac{\partial}{\partial t} = 0\right)$ dieser Gleichung welche läutet:

$$N(\varepsilon) = C e^{-\frac{(\varepsilon-\varepsilon_0)^2 \beta}{D}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

Wir wollen jetzt zeigen dass diese Lösung weitgehend analog zu einer kanonischen Gesamtheit ist.

Dazu schreiben wir nach GIBBS für die Zahl der Systeme mit der Energie zwischen $\varepsilon + d\varepsilon$ in der kanonischen Gesamtheit

$$N(\varepsilon)\, d\varepsilon = e^{\frac{\psi-\varepsilon}{\theta}+\varphi}\, d\varepsilon.$$

mit

$$M = \int e^{\frac{\psi-\varepsilon}{\theta}+\varphi}\, d\varepsilon.$$

Wenn nun ε_0 die mittlere Energie in der Verteilung ist, definiert durch

$$\varepsilon_0 = \int \varepsilon\, e^{\frac{\psi-\varepsilon}{\theta}+\varphi}\, d\varepsilon$$

kann man schreiben

$$\frac{\psi-\varepsilon}{\theta}+\varphi=\frac{\psi-\varepsilon_0}{\theta}+\frac{\varepsilon_0-\varepsilon}{\theta}+\varphi_0+\left(\frac{d\varphi}{d\varepsilon}\right)_0(\varepsilon-\varepsilon_0)+\frac{1}{2}\left(\frac{d^2\varphi}{d\varepsilon^2}\right)(\varepsilon-\varepsilon_0)^2$$

wobei der Index 0 andeutet dass der Wert für die mittlere Energie zu nehmen ist. Nun hat GIBBS gezeigt dass

$$\left(\frac{d\varphi}{d\varepsilon}\right)_0=\frac{1}{\theta}$$

Man kann also die Zahl der Systeme im Bereich zwischen ε und $\varepsilon+d\varepsilon$ darstellen durch

$$N(\varepsilon)\,d\varepsilon=e^{\frac{\psi-\varepsilon_0}{\theta}+\varphi_0}\,e^{\frac{1}{2}\left(\frac{d^2\varphi}{d\varepsilon^2}\right)_0(\varepsilon_0-\varepsilon_0)^2},$$

woraus sich ergibt dass die stationäre Lösing unserer Differentialgleichung ... 2 eine kanonische Gesamtheit darstellt, wobei

$$-\frac{\beta}{D}=\frac{1}{2}\left(\frac{d^2\varphi}{d\varepsilon^2}\right)_0$$

zu setzen ist. Integration der Gleichung 3 nach ε liefert $\frac{D}{\beta}=\overline{(\varepsilon-\varepsilon_0)^2}$ was mit dem GIBBSschen Resultate für $\left(\frac{d^2\varphi}{d\varepsilon^2}\right)_0$ übereinstimmt.

Hat man nun zur Zeit t eine willkürliche Verteilung so wird sich diese Verteilung nach der generalisierten Diffusionsgleichung ändern. Wie man leicht einsieht bedeutet das, dass eine willkürliche Gesamtheit im Laufe der Zeit in eine kanonische Verteilung übergehen wird. Denn die Dichte die zur Zeit t in jedem Volumen-element des Phasenraumes existiert, wird sich nach der Zackenlösung der generalisierten Diffusionsgleichung derart über den Raum ausbreiten dass sie in die Verteilung ... 3 übergeht. Als Relaxationszeit wird $\frac{1}{\beta}$ eine Rolle spielen.

Es ist also möglich die GIBBSsche Hypothese der kanonischen Gesamtheit durch den Ansatz .. 1 zu ersetzen. Weil dieser Ansatz sich nicht nur auf den stationären Zustand bezieht ist damit die Möglichkeit geschaffen auch Systeme welche sich nicht im Gleichgewicht befinden im Kreise der statistischen Behandlungsweise hineinzuziehen.

In einer zweiten Veröffentlichung möchten wir darüber einige Anwendungen bringen und auch auf eine analoge Betrachtungsweise für die mikrokanonische Gesamtheit berichten.

Utrecht Physisch Laboratorium.
Bussum.

Hydrodynamics. — *On the application of statistical mechanics to the theory of turbulent fluid motion.* IV. By J. M. BURGERS. (Mededeeling N⁰. 26 uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hoogeschool te Delft).

(Communicated at the meeting of March 25, 1933).

1. *Introduction.*

The present paper is a continuation of some investigations which have been published in 1929[1]); in particular it intends to give a further development to the formulation of the problem, indicated in § 11 of Part III. In connection with the latter point it is necessary to recollect that the point of view taken in constructing an equation which should govern the various possible "states" of the turbulent motion, had been shifted during the course of this former work, so that it consisted of three sections, the first one formed by Parts I and II (§§ 1—7), the second one by §§ 8—10 of Part III, and the third one by § 11[2]). On account of this circumstance it may be useful to summarize the leading assumptions upon which the statistical considerations were founded, the more so, as it might be asked what is the purpose of introducing a rather abstract and complicated system of statistical formulae, while in the last three years such remarkable successes have been obtained by the theories put forth by PRANDTL and by VON KARMAN[3]). A few words may be said about these latter theories first.

The starting point of the deductions given by PRANDTL and by VON KARMAN is that the shearing stress acting between two adjacent layers may be represented by the formula:

$$\tau = \varrho\, l^2 \left(\frac{dU}{dy}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (*)$$

where U is the velocity of the mean motion, considered as a function of the distance y from the wall, while l is a quantity of the nature of a

1) Parts I—III: these Proceedings **32**, p. 414, 643, 818, 1929.

2) Two other papers (one published in the "Vorträge a. d. Gebiete der Aerodynamik usw. Aachen 1929", p. 21, and the other in the Journal of applied physics (Moscow) **7**. 3, p. 3, 1930) mainly reproduce the treatment given in the first section, though with some changes in the notation and in the discussion of the formulae describing the statistical distribution.

3) A clear exposition has been given by PRANDTL in a paper titled: Neuere Ergebnisse der Turbulenzforschung, Zeitschr. d. Ver. deutsch. Ing. **77**, p. 105, 1933. For VON KARMAN's deductions see: Nachr. Ges. d. Wiss. Göttingen, Math. phys. Klasse 1930, p. 58, and other papers (references in PRANDTL's paper).

length, which is called the "Mischungsweg" and is a measure for the distances travelled by small quantities of fluid in a direction perpendicular to that of the mean motion. An assumption then is introduced which gives some information about the magnitude of this quantity l; VON KARMAN proposed: $l = \varkappa\, (dU/dy) / (d^2U/dy^2)$, while PRANDTL assumed the formula: $l = \varkappa y$, $\varkappa$ in both cases being a numerical constant. With the latter formula equation (*) gives: $\varkappa y\, (dU/dy) = \sqrt{\tau/\varrho}$; taking the case of the flow in the neighbourhood of the wall, where τ may be considered as approximately independent of y, it follows that:

$$U = \frac{1}{\varkappa}\sqrt{\frac{\tau}{\varrho}}\, lg\, \frac{y}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (**)$$

a being a constant of integration. This constant, which again is of the nature of a length, must be a function of the dimensions of the asperities on the wall and of the quantity $\nu\sqrt{\varrho/\tau}$, which also is of the nature of a length and introduces the influence of the viscosity of the fluid. By means of some simple assumptions it appears possible to obtain formulae for the distribution of the velocity of the mean motion and for the resistance, both for the case of rough walls and for that of smooth walls, which are in very good agreement with the latest experimental results, especially with those found by NIKURADSE. In the particular case of a smooth wall the relation must be of the form:

$$a = c\nu\sqrt{\varrho/\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (***)$$

The values of the numerical constants $\varkappa$ and c are deduced from the comparison with the experimental data.

This theory thus has a very inspiring character and yields many interesting and beautiful results. Still it leaves unsettled certain important points, as for instance the structure of the turbulent motion, and the connection between the motion in the central "core" of the field, and the boundary layers in the immediate vicinity of the walls. In the "core" the shearing stress is wholly determined by the turbulent motions, and the fluid can be considered as behaving if it were an ideal fluid; in the boundary layers turbulence disappears and makes place for laminar viscous flow. But in some way the motions in these regions must influence each other. Moreover, this theory does not give any information about the balance of energy.

The idea underlying the present paper and its predecessors, on the contrary, is that the intensity and the distribution of the turbulent motion over the various regions of the field is governed primarily by the so-called "dissipation condition", as given by REYNOLDS and by LORENTZ. If we consider the motion of a fluid through a tube or between parallel walls, and suppose that the quantity Q which in unit time crosses a perpendicular section, is given, then the appearance of turbulent motion, as is well known, will cause an increase of resistance.

Hence an increased pressure drop is required to force the fluid through the tube, and this implies an increased expenditure of energy. At the same time the rate of dissipation of energy in the fluid is increased, partly on account of the circumstance that the relative motions superposed on the mean motion are themselves subject to a rather intense dissipation, partly also in consequence of the fact that the distribution of the mean motion over the section of the tube is changed, and that a considerably increased velocity gradient has appeared in the immediate neighbourhood of the walls. The latter circumstance makes that the turbulence cannot be increased beyond a certain intensity, as then the total dissipation would surpass the input of energy.

This is the idea which has been put forth by REYNOLDS in his paper on the "Dynamical theory of incompressible viscous fluids and the determination of the criterion", published in 1895 [1]). In a paper by the present author from 1923 [2]) it was applied in order to arrive at some preliminary results: by constructing certain systems, representing special types of relative motion, it was possible to obtain a definite value of the resistance for each system. This was done by means of equation (17) of the paper mentioned, which is closely related to eq. (79) of REYNOLDS' paper and gave a relation between a quantity σ proportional in a certain way to the square of the amplitude of the relative motion, and some quantities connected with the structure of the vortices assumed, and with their distribution over the field of flow. As the value of the resistance coefficient calculated depended upon the type of relative motion chosen, a definite result could not be arrived at without further assumptions. Following a suggestion by VON KARMAN it was tried to estimate to what values the resistance coefficient might be increased by changing some parameters characterizing the distribution of the vortices. Then, however, this way of attacking the problem was dropped, as it was clear that the method of constructing special systems of relative motion had to be replaced by some statistical algorithm.

As has been pointed out in § 1 of Part I, the application of statistical methods requires the introduction of some principle which enables us to define the "probabilities" of the various "states" of the motion. In order to arrive at such a principle it has been supposed that the field of motion, say a certain length of the tube or of the space between the parallel walls, is observed at M instants, separated by equal intervals of time, M being a very large number. A microscopically described "state" of the system then is defined by the sequence of the M pictures obtained in this way. On the other hand we assume that we are in possession of a classification of all possible modes of motion of the system under consideration, a classification which must satisfy the condition that all the enumerated modes of motion are of equal weight. The construction

[1]) O. REYNOLDS, Papers on mechanical and physical subjects, Vol. II, p. 575—577.

[2]) These Proceedings **26**, p. 582, 1923.

of such an enumeration appears a very difficult matter for the case of threedimensional motion; in the case of twodimensional motion, however, it can be done with the aid of the stream function, in the way as was indicated in § 2 of Part I, by introducing a certain multidimensional space (the "ξ-space"). Every point of this space denotes a possible mode of motion. If this space is divided into cells of equal volume, having sufficiently small dimensions, we may take the centers of gravity of these cells as the representatives of an enumerable series of modes of motion, which all have to be considered as possessing the same "weight". It has been mentioned at the end of § 11 of Part III that we may also develop the stream function according to a series of normal solutions ("Eigenlösungen") of a differential equation; then the amplitudes or coefficients with which the various normal solutions occur in the development can be introduced as coordinates into the "ξ-space", and thus can be used as a framework for the construction of the cells in this space. [1])

If the various modes of motion are numbered consecutively 1, 2, 3, . . . , we can determine the number of times each one of them occurs in the sequence of M pictures, which constituted a microscopically described state of the system. For a statistical description these numbers n_1, n_2, n_3, ... only are relevant, the order in which the various modes occur in the sequence having no influence upon mean values. Hence a "statistically described state" is uniquely determined by the set of numbers n_1, n_2, n_3, ... To a statistically described state there belong $W = M!/(n_1!\, n_2!\, n_3! \ldots .)$ microscopically described states or sequences.

[1]) Problems and methods connected with the representation of an aggregate of functions by means of a system of coordinates in a multidimensional space have been considered in various treatises, to which the reader may be referred. In the present case a complication perhaps might arise, due to the fact, that we must represent either the stream function Ψ for the actual motion, or introduce separate stream functions for the mean motion and for the relative motion. However, as we shall see below in § 3, we provisionally limit ourselves to an investigation of the various types of the relative motion, which are described by a stream function ψ; the mean motion will then be left out of account. As the stream function ψ must satisfy the boundary conditions: $\psi = 0$, $\partial \psi / \partial y = 0$ both for $y = -\frac{1}{2}$ and for $y = +\frac{1}{2}$, the differential equation to which allusion has been made in the text in connection with the construction of a system of normal functions, must be of the fourth order.

The restriction of the investigations to the twodimensional case may introduce special features, which are not to be found in actual turbulent motion. It has been pointed out indeed by TAYLOR and by PRANDTL that certain differences are to be expected between the cases of twodimensional and of threedimensional motion in the mechanism of turbulent friction and of the transport of heat, and on the other hand, as has been indicated by RIABOUCHINSKY and by PRANDTL, it is highly probable that in the phenomena of ordinary turbulence vortices having their axes parallel, or nearly parallel, to the direction of the mean flow, play an important part (see the paper by PRANDTL, mentioned before; further: G. I. TAYLOR, Proc. Roy. Soc. (A) 135, p. 685, 1932, and D. RIABOUCHINSKY, Verhand. 2. Intern. Kongr. f. techn. Mechanik, Zürich 1926, p. 517, and other papers). But the difficulties of the treatment of the threedimensional case are so great, that provisionally we must restrict ourselves to the problem of motions in two dimensions only.

Now the differences between the various formulations of the statistical problem to which allusion was made at the beginning of the present paper, are connected with the way in which were defined certain mean values occurring in the dissipation condition. The way of attacking the problem applied in the next § seems to be the most natural one; as mentioned it is the formulation arrived at in § 11 of Part III, but for the sake of clearness it will be repeated here in a somewhat more direct form. It is characterized by the circumstance that only quantities defined for instantaneously observed modes of motion are considered, while the circumstance that the motion must have a stationary character over the long run of time is introduced explicitly as a special condition which must be satisfied by the sequence. In this way such questions as arose in connection with the other formulations, are got rid of.

2. *Derivation of the statistical formulae.*

We start from the ordinary equations for twodimensional motion, which need not be repeated in full. Taking the case of the motion between two fixed parallel walls, we shall introduce nondimensional variables by dividing all lengths by h, the breadth of the channel, and all velocities by V_0, the mean of the velocity taken over a perpendicular section; then the pressure gradients must be divided by $\varrho\, V_0^2/h$, while the viscosity must be replaced by the inverse REYNOLDS' number $R^{-1} = \mu/\varrho\, V_0\, h$. The first one of the equations of motion thus takes the form:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{R}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \quad . \quad . \quad . \quad (1)$$

In order to arrive at an equation which instead of the quantity $-\partial p/\partial x$ contains a quantity which is independent of x and y, we take the mean of (1) with respect to x; that is to say, we integrate (1) with respect to x over a great length L, and divide afterwards by L, neglecting at the same time quantities of the order L^{-1}, as they can be made as small as we please. Indicating mean values defined in this way by means of a bar, we obtain:

$$\frac{\partial \bar{u}}{\partial t} = -\overline{\frac{\partial p}{\partial x}} - \frac{\partial}{\partial y}\,\overline{uv} + \frac{1}{R}\frac{\partial^2 \bar{u}}{\partial y^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For simplicity we shall write $2\,C$ for $\overline{-\partial p/\partial x}$. By considering the second one of the equations of motion it can be shown that C is independent of y; in fact if C were a function of y, pressure differences of arbitrary great magnitude in the points of a single perpendicular section of the channel would be obtained for very great values of x, which clearly is impossible. In the usual way we further write: $\bar{u} = U$, $u = U + u'$, $v = v'$ ($\bar{v}$ being zero); then equation (2) assumes the form:

$$\frac{\partial U}{\partial t} = 2\,C - \frac{\partial}{\partial y}\,\overline{(u'\,v')} + \frac{1}{R}\frac{\partial^2 U}{\partial y^2}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This equation shall be used to construct one of the conditions which must be satisfied by a statistically described state of the system. It is evident that on the long run the mean value of $\partial U/\partial t$ taken with respect to the *time* must be zero, if the motion shall be stationary. Hence if we indicate by σ_m the value of the right hand member of eq. (3) for the mode of motion numbered m in our enumeration of possible modes of motion, the sequence of M observed modes of motion constituting a state of the system must satisfy the condition:

$$\sum_m n_m \sigma_m = 0 \quad \ldots \ldots \ldots \ldots \quad (4)$$

together with:

$$\sum_m n_m = M \quad \ldots \ldots \ldots \ldots \quad (5)$$

The condition (4) must be satisfied for all values of y. We shall not, however, require that it is satisfied also for y exactly equal to $-\frac{1}{2}$ or to $+\frac{1}{2}$, that is at the walls.

In the former work we have always eliminated C from equation (3), by multiplying it with y and integrating over the breadth of the channel, making use of the relation $\int U\,dy = 1$ [1]. This procedure, which leads to a somewhat complicated form of the equation, can be omitted, however, as it is much simpler to eliminate C afterwards by a formal device

The second condition which governs the states of the system, is the dissipation condition. This can be obtained in the simplest way by observing that at any given instant the mean value (per unit of length of the channel) of the energy put into the system is given by the mean pressure drop, multiplied into the total rate of flow, that is in our nondimensional notation by $2\,C$. The rate of dissipation, on the other hand, is determined by the mean square of the vorticity, integrated over the breadth of the channel. Indicating for shortness the mean square of the vorticity of the relative motion by

$$\bar{z} = \overline{\left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right)^2}$$

and taking notice at the same time of the vorticity of the mean motion, which is given by $-\partial U/\partial y$, we arrive at the following equation for the rate of increase of the kinetic energy of the flow per unit of length of the channel:

$$\frac{dE}{dt} = 2\,C - \frac{1}{R}\int dy \left\{\bar{z} + \left(\frac{\partial U}{\partial y}\right)^2\right\} \quad \ldots \ldots \quad (6)$$

The value of the right hand member of this equation for the type of

---

[1]) All integrals with respect to y are to be extended from $y = -\frac{1}{2}$ till $y = +\frac{1}{2}$.

motion numbered m, will be indicated by ε_m. Then the second condition which must be satisfied by any state of the system is:

$$\sum_m n_m \, \varepsilon_m = 0 \quad \ldots \ldots \ldots \ldots \quad (7)$$

which condition applies to the whole field at once.

Now introducing the "weight" of a statistically described state, as given by the number of permutations W, and reasoning in the ordinary way, the statistical mean values of the n_m, taken over a great number of sequences, each consisting of M observations, can be calculated. They will be denoted by $\overline{n_m}$ and are given by the formula:

$$\overline{n_m} = e^{c + \beta \left(\varepsilon_m - \int dy \, \lambda \, \sigma_m\right)} \quad \ldots \ldots \ldots \ldots \quad (8)$$

where c and β are numerical constants, while λ is a function of y. The constants c and β are determined by the condition that (5) and (8) must be satisfied; the function λ is determined by the condition that eq. (4) shall be fulfilled for all values of y satisfying the inequalities: $-\frac{1}{2} < y < +\frac{1}{2}$. On account of the circumstance that we do not require that (4) shall be fulfilled at $y = -\frac{1}{2}$ or $+\frac{1}{2}$, we assume that λ is zero at $y = -\frac{1}{2}$ and at $y = +\frac{1}{2}$. Further in order to eliminate the mean pressure drop $2\,C$ from the exponent in (8) we subject λ to the condition [1]):

$$\int \lambda \, dy = 1 \, . \quad \ldots \ldots \ldots \ldots \quad (9)$$

The integral $\int dy \, \lambda \, \sigma_m$ in (8) can be transformed by means of partial integration. Omitting the details we finally obtain:

$$\overline{n_m} = e^{c + \beta \int dy \left[\frac{1}{R} \left\{ \frac{\partial U}{\partial y} \frac{d\lambda}{dy} - \left(\frac{\partial U}{\partial y} \right)^2 \right\} - \overline{u' v'} \frac{d\lambda}{dy} - \frac{\overline{z}}{R} \right]_m} \quad \ldots \ldots \quad (10)$$

where the index m indicates that the quantity between the square brackets must be calculated for the mode of motion numbered m.

3. *Device for the introduction of a series of normal functions for the development of the stream function ψ of the relative motion.*

The conditions which determine the constants c and β and the function λ occurring in the exponent of e in (10) can be expressed by introducing a function Φ defined by:

$$\Phi = \sum_m e^{c + \beta \int dy \, [\ldots]_m} \quad \ldots \ldots \ldots \ldots \quad (11)$$

where the summation with respect to m must be extended over all modes of motion, or in other words, over the whole of the system of cells into

---

[1]) The function φ^* introduced by eq. (75) of Part III is connected with λ in such a way, that $R\,\varphi^*$ corresponds to $d\lambda/dy$; φ^* satisfies the relation: $R \int \varphi^* \, y \, dy = -1$.

which the "ξ-space" has been divided (this summation may be replaced by an integration over the "ξ-space"). Then the function Φ must take the value M, and at the same time it must be stationary with respect to arbitrary variations of β and of the function λ, the latter, however, being subject to eq. (9) and to the condition that $\lambda = 0$ for $y = \pm \frac{1}{2}$.

Now although the values of $-\overline{u' v'}$, etc. must be considered as known for every point of the "ξ-space", the evaluation of an integral of this kind presents very great difficulties. The only thing we can do is to try to get some insight into the meaning of the integral and of the distribution function defined by eq. (10) by means of approximate methods.

We shall begin by giving some attention to the problem posed by the distribution of the relative motions. This can be done in consequence of the circumstance that the exponent in (10) can be divided into two parts, one depending on the mean motion, the other one depending on the relative motion. Omitting a constant factor we thus have to consider the formula:

$$\overline{n_m} = e^{\beta \int dy \left[-\overline{u' v'} \frac{d\lambda}{dy} - \frac{\bar{z}}{R} \right]_m} \quad \ldots \ldots \ldots \quad (12)$$

The possibility of evaluating the integral in the exponent depends upon the method we shall choose in representing the various modes of motion 1, 2, 3, ... As has been mentioned before we may base the enumeration of the modes of motion upon a system of coordinates in the "ξ-space", derived from a system of normal functions of a differential equation of the fourth order. Pursuing this idea we shall assume for a moment that some function λ has been chosen. Then we shall investigate which stream functions ψ for the relative motion give a maximum value to the fraction:

$$\frac{-\int dy\, \overline{u' v'} \frac{d\lambda}{dy}}{\frac{1}{R} \int dy\, \bar{z}} \quad \ldots \ldots \ldots \quad (13)$$

This brings us to a type of problems which are well known from the theoretical researches on the stability of laminar motion. If we put:

$$\psi = \chi_I(y) \,.\, \cos ax + \chi_{II}(y) \,.\, \sin ax \,. \quad \ldots \ldots \quad (14)$$

we easily find the following expressions for $-\overline{u' v'}$ and $\bar{z}$[1]):

$$-\overline{u' v'} = \frac{a}{2} (\chi_I' \chi_{II} - \chi_I \chi_{II}') \quad \ldots \ldots \quad (15)$$

$$z = \tfrac{1}{2} (\chi_I'' - a^2 \chi_I)^2 + \tfrac{1}{2} (\chi_{II}'' - a^2 \chi_{II})^2 \quad \ldots \ldots \quad (16)$$

If these expressions are inserted into (13) we are led to a variational problem for the two functions χ_I, χ_{II} (which can be varied independently

---

[1]) In these formulae χ'_I denotes $d\chi_I/dy$, and so on.

of each other); by means of the well known rules we thus are led to a system of two simultaneous differential equations:

$$\left.\begin{aligned} \chi_I^{IV} - 2\,\alpha^2 \chi_I'' + \alpha^4\,\chi_I + \alpha R \Lambda \left(\frac{d\,\lambda}{dy}\,\chi_{II}' + \frac{1}{2}\,\frac{d^2\,\lambda}{dy^2}\,\chi_{II}\right) = 0 \\ \chi_{II}^{IV} - 2\,\alpha^2 \chi_{II}'' + \alpha^4\,\chi_{II} - \alpha R \Lambda \left(\frac{d\,\lambda}{dy}\,\chi_I' + \frac{1}{2}\,\frac{d^2\,\lambda}{dy^2}\,\chi_I\right) = 0 \end{aligned}\right\} \quad \ldots \quad (17)$$

If we write:

$$\chi = \chi_I + i\,\chi_{II} \quad \ldots\ldots\ldots\ldots \quad (18)$$

the equations can be contracted into the single one:

$$\chi^{IV} - 2\,\alpha^2 \chi'' + \alpha^4\,\chi - i\,\alpha R \Lambda \left(\frac{d\,\lambda}{dy}\,\chi' + \frac{1}{2}\,\frac{d^2\,\lambda}{dy^2}\,\chi\right) = 0. \quad \ldots \quad (19)$$

By changing i into $-i$ we obtain the equation to be satisfied by the conjugate function $\bar{\chi} = \chi_I - i\,\chi_{II}$.

The number Λ in eq. (19) is the "characteristic number"; solutions satisfying the boundary conditions:

$$\chi = \chi' = 0 \text{ for } y = -\frac{1}{2},\ y = +\frac{1}{2} \quad \ldots\ldots \quad (20)$$

can be obtained only for a definite series of values of this parameter. If the "characteristic values" are denoted by Λ_k, and if the corresponding "normal solutions" are written χ_k, it can be demonstrated:

1. that all Λ_k are real;

2. that any two functions χ_k, χ_l, belonging to different characteristic numbers Λ_k, Λ_l, satisfy the relations:

$$\int dy\,\frac{d\lambda}{dy}\,(\chi_k'\,\bar{\chi}_l - \chi_k\,\bar{\chi}_l') = 0 \quad \ldots\ldots\ldots \quad (21)$$

$$\int dy\,(\chi_k'' - \alpha^2\,\chi_k)\,(\bar{\chi}_l'' - \alpha^2\,\bar{\chi}_l) = 0. \quad \ldots\ldots\ldots \quad (22)$$

In order to fix the arbitrary constant occurring in any normal function, we take:

$$\int dy\,\frac{d\lambda}{dy}\,(\chi_k'\,\bar{\chi}_k - \chi_k\,\bar{\chi}_k') = -\frac{2i\,\alpha^2}{R} \quad \ldots\ldots \quad (23)$$

Then at the same time it is found that:

$$\int dy\,(\chi_k'' - \alpha^2\,\chi_k)\,(\bar{\chi}_k'' - \alpha^2\,\bar{\chi}_k) = \Lambda_k\,\alpha^3 \quad \ldots\ldots \quad (24)$$

The application of a system of functions χ_k to the reduction of (12) shall be considered in a further paper.

Chemistry. — *Osmotic systems in which non-diffusing substances may occur also.* IV. By F. A. H. SCHREINEMAKERS.

(Communicated at the meeting of March 25, 1933).

IV. *Systems with elastic walls and membranes.*

In the preceding communications we discussed osmotic systems, in which the pressure remains constant on both sides of the membrane, or in which each of the two liquids was enclosed in a space of constant volume; further we assumed that the membrane was inextensible. As, however, Nature has osmotic systems enclosed in spaces with extending walls and membranes, we shall also briefly discuss these systems.

We now take an osmotic system as in fig. 1, in which for the present we overlook the letters P_0 and P'_0. On the left side of the membrane, which

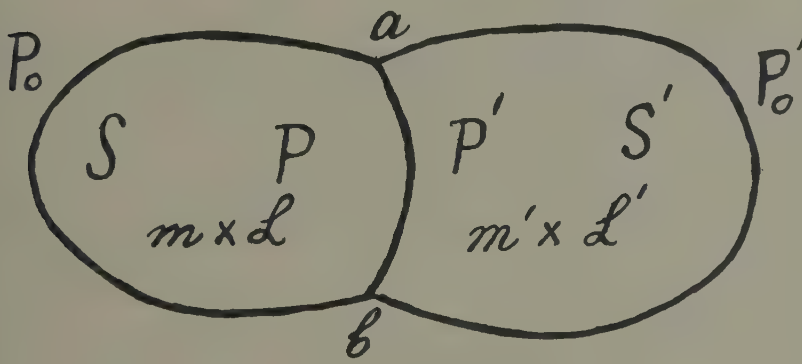


Fig. 1

is represented by the curved line *ab*, is a space S; on the right side is a space S'.

By the wall of space S (or S') the entire wall of this space, except the part formed by the membrane, is meant; we now assume that the walls of these spaces and the membrane are elastically extensible.

We imagine space S filled with m quantities of a liquid L under the pressure P, space S' with m' quantities of a liquid L' under the pressure P'.

Of course these spaces S and S' may have all sorts of shapes, which depends on their walls being completely or only partially extensible, on the elasticity in different directions, etc. It may be imagined among other things that the walls extend principally in one direction only, as may be the case with growing cells of plants.

For the sake of simplicity we now shall represent the system of fig. 1 by:

$$ {}^{r}_{s}\Big(m \times L\,(d+n)_P\Big)^{a}_{b}\, m' \times L'\,(d+n')_{P'}\Big)^{r'}_{s'} \quad . \quad . \quad . \quad . \quad (1a) $$

in which the curvature of the lines *ab*, *rs* and *r′s′* indicate that the membrane and the walls of the spaces S and S' are elastically extensible; inextensible membranes and walls will be represented by straight lines.

If we imagine the wall of one of the spaces e.g. of S, inextensible, then we get the system:

$$ {}^{r}_{s}\Big|\, m \times L\,(d+n)_P\Big)^{a}_{b}\, m' \times L'\,(d+n')_{P'}\Big)^{r'}_{s'} \quad . \quad . \quad . \quad . \quad (2a) $$

When the walls of the two spaces become inextensible, we get:

$$ {}^{r}_{s}\Big|\, m \times L\,(d+n)_P\Big)^{a}_{b}\, m' \times L'\,(d+n')_{P'}\Big|^{r'}_{s'} \quad . \quad . \quad . \quad . \quad (3a) $$

We now may imagine the membrane to be inextensible also in each of the systems (1*a*)—(3*a*); then we shall call them (1*b*)—(3*b*).

We are now going to call a variable space, a space, the volume of which can change because of the elasticity of wall or membrane or of both; a space, the volume of which does not change, is called invariant. Then each of the systems (1*a*), (2*a*), (3*a*) and (1*b*) has two variable spaces; system (2*b*) has one variable and one invariant space and system (3*b*) has two invariant spaces.

We now take a variable space with a given wall and a given membrane, so that between the volume and the pressure P within this space a relation

$$ \varphi\,(P \,.\, V) = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4) $$

exists; with every pressure P the volume V and reversally with every volume V the pressure P will then be defined. We shall call such a variable space: "definite variable".

When there is besides an external pressure on the wall of this space, it will also occur in (4); the same obtains for the pressure which the liquid of the other space brings to bear on the membrane, when it is extensible. The function φ, which contains among other things also the magnitudes, determining the elasticity of space or membrane or of both, is generally unknown; we may assume, however, that the volume of this space will increase when the pressure inside this space increases.

When in a system the osmosis is over and this system has consequently attained on equilibrium, then, as we have mentioned already in preceding Comm., the d equations:

$$ O\,A = (O\,A)' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5) $$

obtain, expressing that each of the d diffusing substances has the same *O.A.* viz. the same osmotic attraction on both sides of the membrane. In order to deduce this for the general case of system (1a), we imagine this represented again by fig. 1; we assume here that an external pressure P_0 is brought to bear on the wall of space S and an external pressure P_0' on the wall of space S'.

When dm quantities of one of the diffusing substances travel through the membrane towards the left, then the volumes V and V' of each of the two spaces will change a little; now we assume:

1. the wall of space S moves outwards over a volume dv; by this I mean that the volume between the new and the old position of this wall is dv.
2. the membrane moves towards the right over a volume dv_1.
3. the wall of the space S' moves inwards over a volume dv'.

Then the volumina V and V' increase with

$$dV = dv + dv_1 \quad \text{and} \quad dV' = -(dv_1 + dv') \quad . \quad . \quad . \quad . \quad (6)$$

If we now represent the entire free energy of the two liquids by ψ and ψ', that of the two walls by ψ_r and ψ'_r and that of the membrane by ψ_m, then

$$d(\psi + \psi' + \psi_r + \psi'_r + \psi_m) + P_0\, dv - P'_0\, dv' = 0 \quad . \quad . \quad . \quad (7)$$

must be satisfied, in which the two last terms indicate the external work performed by the system. We now have:

$$d\psi = \left(\frac{\partial\psi}{\partial m}\right)_V dm + \left(\frac{\partial\psi}{\partial V}\right)_m dV = \left(\frac{\partial\psi}{\partial m}\right)_V dm - P(dv + dv_1) \quad . \quad . \quad (8)$$

As the liquid of space S' gives off the dm quantities of the diffusing substance, it follows:

$$d\psi' = -\left(\frac{\partial\psi'}{\partial m}\right)_V dm + P'(dv_1 + dv') \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If we take into consideration the work, performed on the wall of space S and through the wall of space S', we find:

$$d\psi_r = (P - P_0)\, dv \quad \text{and} \quad d\psi'_r = -(P' - P'_0)\, dv' \quad . \quad . \quad . \quad (10)$$

For the change in the free energy of the membrane we find:

$$d\psi_m = (P - P')\, dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If we substitute (8)—(11) in (7) then we find that

$$\left(\frac{\partial\psi}{\partial m}\right)_V = \left(\frac{\partial\psi'}{\partial m}\right)_{V'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

must be satisfied.

This means that the diffusing substance must have the same thermodynamical potential in both liquids. As this obtains for each of the d diffusing substances and as the $O.A.$ of a substance is equal to its potential with the negative sign, it appears that for equilibrium (1) the d equations (5) are valid. So the same also obtains for the equilibria, which can be deduced from (1).

Liquid L contains $d+n$ substances, so that its composition is determined by $d+n-1$ concentrations; if we add to this also the quantity m, the pressure P and the volume V, then the left-side liquid contains, therefore, $d+n+2$ variables. As liquid L' contains $d+n'$ substances, both liquids will contain together

$$2d+n+n'+4 \text{ variables} \quad \ldots\ldots \quad (13)$$

If we represent the volume of one quantity of liquids L and L' by v and v', we have:

$$V=mv \quad \text{and} \quad V'=m'v' \quad \ldots\ldots \quad (14)$$

as $v(v')$ depends upon the composition and the pressure of liquid $L(L')$, both these equations contain together all variables.

Now we imagine that the spaces of the systems are either "definite variable" or "invariant". Then two equations (4) obtain for the systems (1a), (2a), (3a) and (1b), which will then have two "definite variable" spaces. For system (2b) in which S' is "definite variable" and S is "invariant", then one of the relations (4) will obtain and $V=$constant. For system (3b), in which both spaces are invariant, $V=$constant and $V'=$constant obtain. So for each of these systems two relations obtain, which we shall call (14a).

If we now imagine that these systems have attained equilibrium, so that the d equations (5) are valid too, then $d+4$ relations exist between the variables, therefore; from this it follows:

A. a free Q-equilibrium with "definite variable" or "invariant" spaces has:

$$d+n+n' \text{ freedoms} \quad \ldots\ldots\ldots \quad (15)$$

Here, just as in the preceding communications, we mean by a free Q-equilibrium an equilibrium in which the quantity of each of the substances present can still be changed arbitrarily.

In a Def. Q-equilibrium, however, the quantity of each of the diffusing and non-diffusing substances has a definite invariable value. Then there are d relations, expressing that the quantity of each of the d diffusing substances has a given value; n relations expressing the same for the n non-diffusing substances on the left side and n' relations, expressing the same for the n' non-diffusing substances on the right side of the membrane

(compare also the preceding communications). As there still exist, therefore

$$d + n + n' \text{ relations,}$$

it follows:

B. a Def. Q-equilibrium with definite variable or invariant spaces has no freedoms. The volumina V and V', the pressures P and P', the quantities m and m' and the compositions of the two liquids L and L' are completely defined.

In Comm. II and III the influence, which the change in the permeability of a membrane can have upon the state of an equilibrium, has been discussed. If we now consider the change in the elasticity of the membrane or of the walls as well, we find:

C. in a Def. Q-equilibrium with definite variable or invariant spaces every change in the membrane, by which this becomes more or m.l. permeable, and every change in the elasticity of the membrane or of the walls also causes a change in the e-state; each of the changes is then followed by an osmosis until the new e-state has set in.

Here we mean by e-state the state of equilibrium of the system.

We now take an osmotic system:

$$\left.{}^{r}_{s}\!\left(m_0 \times L_0\right)^{a}_{a} inv.\, L_i\, (d + n')_{P_i} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

in which on the right side of the membrane ab is an invariant liquid i under the pressure P_i; on the left side is a definite variable space S filled with m_0 quantities of a liquid L_0; we assume that this liquid contains n non-diffusing and an arbitrary number of diffusing substances. If we now leave this system alone, the n non-diffusing substances will of course remain in space S; system (16) then passes into an equilibrium:

$$\left.{}^{r}_{s}\!\left(m \times L\, (d + n)_P\right)^{a}_{b} inv.\, L_i\, (d + n')_{P_i} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

in which liquid L now contains the same d diffusing substances as liquid i. If namely one or more of these d substances were not yet present in L_0, they would certainly enter into it during the osmosis. If in L_0 other diffusing substances were present besides those in liquid i, then they will divide themselves between the two liquids; as, however, liquid i is kept invariant, these substances are being taken away continuously, so that at last they will disappear from the system.

In nature such systems may occur e.g. when some liquid or other flows along the membrane keeping practically the same composition, and when space S has been filled with a tissue; the film in which this tissue is enclosed then forms the wall of this space.

It is clear that (7) cannot be a Def. Q-equilibrium (compare also Comm.

III); here namely we can only consider the quantity of each of the n non-diffusing substances as given; we now shall call an equilibrium, containing a definite quantity of each of the n non-diffusing substances, a Def. $Q.$ $n.$-equilibrium. We now find:

D. a Def. $Q.n.$-equilibrium with one definite variable or invariant space and with an invariant liquid has no freedoms; each change in the membrane, by which this becomes more or m.l. permeable, and each change in the elasticity of membrane or wall also causes a change in the e-state of the space; every change is followed by an osmosis until the new e-state has set in.

In order to illustrate the preceding considerations, we take the equilibrium

$$ {}^{r}_{s}\Big(\, m \times L\,(W+X)_{P} \,\Big|^{a}_{b}\; inv.\, L_i\,(W+X+\overline{Y})_{P_i} \quad . \quad . \quad . \quad (18)$$

with a definite variable space S; of course the membrane ab may also be supposed to be extensible here. The membrane is, as has been indicated by a horizontal dash over Y, impermeable for Y and, therefore, permeable for W and X only; from this it follows $P < P_i$.

When the membrane now becomes permeable for Y too and hence for all substances, a new equilibrium can arise only then, when the two liquids will get the same pressure and composition; so (18) passes into:

$$ {}^{r}_{s}\Big(\, m_1 \times L_i\,(W+X+Y)_{P_i} \,\Big|^{a}_{b}\; inv\, L_i\,(W+X+Y)_{P_i} \quad . \quad . \quad (19)$$

During the transition from (18) into (19) not only Y but also water and X will pass through the membrane. As the pressure within space S in (19) is now greater than in (18) (viz. $P_i > P$), the volume of S will increase at the transition of (18) into (19).

If, however, we take the equilibrium

$$ {}^{r}_{s}\Big(\, m \times L\,(W+X+\overline{Y})_{P} \,\Big|\; inv\, L_i\,(W+X)_{P_i} \quad . \quad . \quad . \quad (20)$$

then is $P > P_i$. When the membrane now becomes permeable for Y also, then (20) passes into:

$$ {}^{r}_{s}\Big(\, m_1 \times L_i\,(W+X)_{P_i} \,\Big|\; inv\, L_i\,(W+X)_{P_i} \quad . \quad . \quad . \quad . \quad (21)$$

During this transition the volume of space S now will decrease.

If in (18) and (20) we imagine instead of space S a tissue enclosed by an elastic film, then at the transition of (18) into (19) this tissue will swell to a definite volume and perhaps it will burst. During the transition from (20) into (21) this tissue will contract to a definite volume. The same

will take place also when we bring these tissues into such a large quantity of liquid *i*, that the composition of this liquid will practically change only a little.

We may imagine also that the wall of a space partly loses its elasticity through some influence or other; then this wall will put a smaller pressure on the liquid at the same volume; we may imagine also that at the same time the change in the permeability of the membrane discussed above, occurs as well. A closer consideration of these cases is left to the reader.

(To be continued).

Leiden, Lab. of Inorg. Chemistry.

Chemistry. — *The Exact Measurement of Specific Heats at Higher Temperatures.* XI. *On the Variability of the Specific Heats of Fused and Solidified Silver under different Circumstances.* By

F. M. JAEGER, E. ROSENBOHM and W. A. VEENSTRA.

(Communicated at the meeting of March 25, 1933).

§ 1. In the course of our previous work on the specific heats of stabilized and preliminarily worked metals, some unexplained pecularities in the behaviour of fused and solidified *silver* [1]) were observed, which made it desirable again to study this metal at higher temperatures more in detail. On repeating our measurements of the mean specific heats $\bar{c}_p$ with a lump of silver *B* fused and solidified in air [2]), but being afterwards preserved at room-temperature during five or six months, — the remarkable fact was stated that all values of $\bar{c}_p$, measured by means of the same instrument as formerly used, proved to be *appreciably diminished*. The deviations from the values previously found were in all cases much greater than could be accounted for by the possible experimental errors (see curve *B'* in Fig. 1). Originally we thought that the stabilized lump of silver, by its long preservation at room-temperature had, in some way or other, returned to a less stable condition, — such as, for instance, was met with in the case of cold-plated and laminated silver (curve *P* in Fig. 1). Soon,

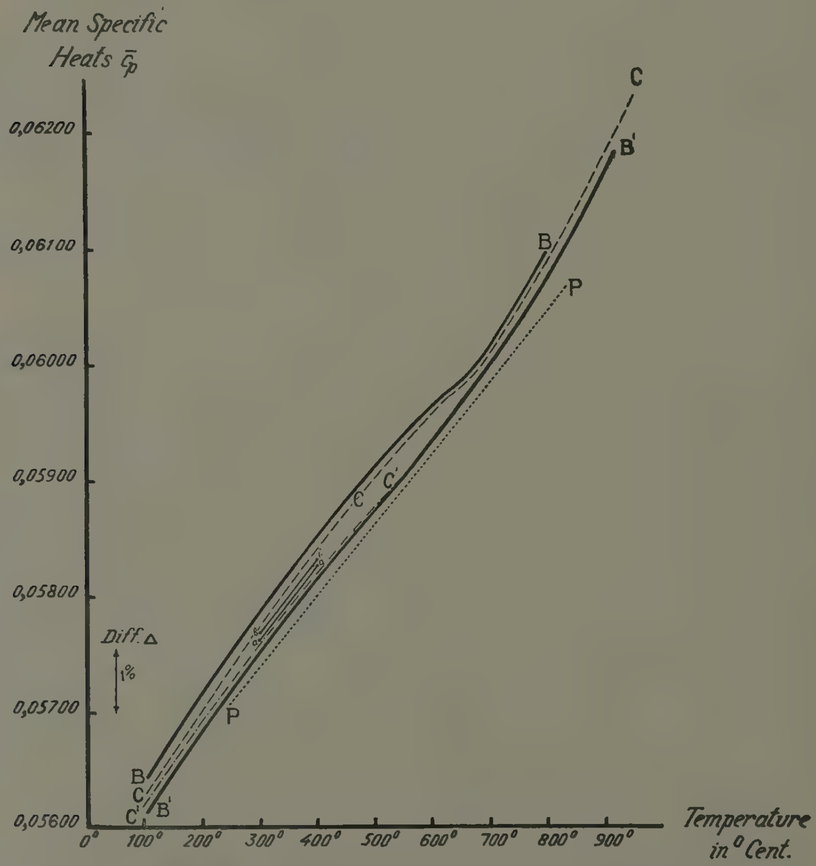
---

[1]) F. M. JAEGER, E. ROSENBOHM and J. A. BOTTEMA, Proceed. R. Acad. Sciences Amsterdam, **35**, (1932), 768; Recueil d. Trav. d. Chim. Pays-Bas, **52**, (1933), 74; conf. Note [13]) at the bottom of page 77, loco cit., on the more rapid increase of $\bar{c}_p$ above 600° – 700° C.

[2]) On page 74, loco cit., it was erroneously mentioned, that the lump of silver used was fused and solidified "in a vacuum": really it occurred *in air*.

however, it became clear to us, that here the situation was quite another one, as may be concluded from the following series of experiments.

Two lumps of silver, fused and solidified *in air*, yielded values of $\bar{c}_p$, which, — within the limits of experimental error, — proved to be identical with those formerly observed in the case of the original mass (conf. curve C in Fig. 1). But if these lumps were remelted and solidified *in a perfect vacuum*, the values of $\bar{c}_p$ at each temperature now proved, — also within the limits of experimental error, — to be *the same* as those obtained in the case of the lump of silver B', which had been preserved during six months (conf. the curve C' in Fig. 1). On the other hand, if this last mentioned lump of silver was, in its turn, heated *in air* at 940° C. for 5 or 10 hours, it again yielded values of $\bar{c}_p$, which were the higher, the longer its heating in air was continued (see curves aa' and bb' in Fig. 1). The limits of experimental error in all these experiments never surpassed 0.1—0.2 % of



Silver: B Original Lump, fused and solidified in Air; B' The same Lump, preserved 5 or 6 Months; C Lump of Silver recently fused and solidified in Air; C' The same, fused and solidified in a Vacuum; aa' and bb', both B', after heating at 940° C. in Air during 5, resp. 10 hours; P cold-plated and laminated Silver.

Fig. 1. Mean Specific Heats of Silver in Different Circumstances.

TABLE I.

Quantities of Heat developed Q_0 and Mean Specific Heats $\bar{c}_p$ between t° and 0° C. for Silver under Different Conditions.

Temperature t in °C.:	Original Lump of Silver B:		The same Lump B', after having been preserved at room-temperature during several months:		Lump of Silver C melted and solidified in Air:		The same Lump C' after being remelted and solidified in a Vacuum:		The Lump of Silver B' after being heated at 940° C. in Air. During 5 hours:		The Lump of Silver B' after being heated at 940° C. in Air. During 10 hours:		Cold plated and laminated Silver P heated in Vacuum:	
	Q_0 in Calor. pro Gr.:	c_p	Q_0 in Calor. pro Gr.:	$\bar{c}_p$:	Q_0 in Calor pro Gr.:	c_p	Q_0 in Calor. pro Gr.:	$\bar{c}_p$:	Q_0 in Calor. pro Gr.:	$\bar{c}_p$:	Q_0 in Calor. pro Gr.:	$\bar{c}_p$:	Q_0 in Calor. pro Gr.:	c_p:
99°.659	—	—			5.612	0.05628	—	—	—	—	—	—	—	—
99.766	—	—				—	5.608	0.05621	—	—	—	—	—	—
99.934	—	—	—	—	5.625	0.05631	—	—	—	—	—	—	—	—
100.045	—	—	—	—	—	—	5.628	0.05626	—	—	—	—	—	—
100.41	—	—	5.634	0.05611	—	—	—	—	—	—	—	—	—	—
100.54			5.643	0.05613	—	—	—	—	—	—	—	—	—	—
100.87	5.690	0.05641	—	—	—	—	—	—	—	—	—	—	—	—
248.1	14.255	0.05744	—	—	—	—	—	—	—	—	—	—	—	—
295.9	17.115	0.05782	—	—	—	—	—	—	—	—	—	—	—	—
299.40	—	—	—	—	17.294	0.05776	—	—	—	—	—	—	—	—
299.83	—	—	—	—	—	—	—	—	—	—	17.292	0.05767	—	—
299.86	—	—	—	—	—	—	—	—	17.279	0.05762	—	—	—	—
299.98	—	—	—	—	—	—	17.256	0.05751	—	—	—	—	—	—
300.0	—	—	17.264	0.05755	—	—	—	—	—	—	—	—	17.1724	0.05724
300.27	—	—	17.285	0.05756	—	—	—	—	—	—	—	—	—	—
396.86	—	—	—	—	23.182	0.05841	—	—	—	—	—	—	—	—
397.2	23.254	0.05853	—	—	—	—	—	—	—	—	—	—	—	—
397.26	—	—	—	—	—	—	—	—	—	—	23.178	0.05834	—	—
397.3	—	—	23.095	0.05813	—	—	—	—	—	—	—	—	—	—
397.44	—	—	—	—	—	—	23.144	0.05816	—	—	—	—	—	—
397.48	—	—	—	—	23.205	0.05838	—	—	—	—	—	—	—	—
397.64	—	—	—	—	—	—	—	—	23.182	0.05830	—	—	—	—
398.6	—	—	23.178	0.05815	—	—	—	—	—	—	—	—	—	—
420.44	—	—	—	—	—	—	—	—	—	—	—	—	24.496	0.05826
516.5	—	—	—	—	—	—	30.395	0.05885	—	—	—	—	—	—
516.6	—	—	30.438	0.05892	—	—	—	—	—	—	—	—	—	—
628.7	—	—	37.545	0.05972	—	—	—	—	—	—	—	—	37.360	0.05942
629.1	—	—	—	—	—	—	37.571	0.05972	—	—	—	—	—	—
629.15	—	—	—	—	37.606	0.05977	—	—	—	—	—	—	—	—
631.4	—	—	—	—	—	—	—	—	—	—	—	—	37.397	0.05923
634.95	—	—	—	—	—	—	—	—	—	—	—	—	37.802	0.05953
640.0	38.306	0.05984		—	—	—	—	—	—	—	—	—	—	—
701.9	—	—	42.271	0.06022	—	—	—	—	—	—	—	—	—	—
749.2	—	—	45.396	0.06059	—	—	—	—	—	—	—	—	—	—
800.7	—	—	—	—	—	—	—	—	—	—	—	—	48.527	0.06060
800.97	—	—	—	—	48.865	0.06101	—	—	—	—	—	—	—	—
801.1	—	—	48.791	0.06090	—	—	—	—	—	—	—	—	—	—
801.2	—	—	—	—	—	—	—	—	—	—	—	—	48.476	0.06050
804.4	49.096	0.06103	—	—	—	—	—	—	—	—	—	—	—	—
849.9	—	—	52.052	0.06124	—	—	—	—	—	—	—	—	—	—
850.87	—	—	—	—	52.254	0.06141	—	—	—	—	—	—	—	—
894.83	—	—	—	—	55.309	0.06181	—	—	—	—	—	—	—	—
895.3	—	—	55.298	0.06176	—	—	—	—	—	—	—	—	—	—
940.3	—	—	—	—	58.553	0.06227	—	—	—	—	—	—	—	—

the values measured; all data obtained are collected in the following table I and graphically represented in Fig. 1.

§ 2. From the experience gathered in this way, it must be concluded, that the variability of the values of $\bar{c}_p$ is immediately connected with *a content of oxygen of the melted and solidified metal.*

The true values of $\bar{c}_p$ for really *oxygen-free*, stabilized silver are the *smaller* ones, represented by the curves B' and C' in Fig. 1, while the *higher* ones (curves B, C and aa' and bb' in Fig. 1) correspond to a mass of the metal which contains a greater or smaller quantity of oxygen, absorbed during the process of fusion in air and evidently partially retained during its solidification, or absorbed by the solid metal heated at 940° C.

TABLE II.

Mean specific Heats $\bar{c}_p$ of stabilized, oxygenfree Silver between 0° and 900° C.

Temperature in 0° C.:	Quantity of Heat Q_0 in Cal. developed by 1 Gr. silver between t^0 and 0° C.:	Mean specific Heat $\bar{c}_p$ between t^0 and 0° C.:	$\bar{c}_p$ calculated from Formula I:	$\bar{c}_p$ calculated from Formula II:
100°.48	5.639	0.05612	—	—
300.14	17.275	0.05755	0.05752	0.05770
397.4	23.120	0.05818	—	—
516.55	30.416	0.05888	0.05898	0.05893
628.9	37.558	0.05972	—	—
701.9	42.271	0.06022	0.06019	—
749.2	45.396	0.06059	0.06050	0.06054
801.1	48.791	0.06090	0.06083	—
849.9	52.052	0.06124	0.06114	0.06129
895.3	55.298	0.06176	0.06142	—
(940.3)	(58.553)	(0.06227)	—	(0.06238)

The absorption of oxygen by the metal evidently causes an increase of $\bar{c}_p$ at each temperature, which is the more considerable, the greater the quantity is of oxygen absorbed by the solid metal. The most accurate values of $\bar{c}_p$ for *oxygen-free*, stabilized silver between 0° C. and 900° C. are collected in table II. Without any doubt the more rapid increase of the *$\bar{c}_p$-t-curve* above 700° C. and its particular ∼-shape, are also connected

with the more accelerated absorption of oxygen by or its diffusion through the heated metal at higher temperatures: all investigations hitherto made[1]) seem to confirm the fact, that *no* allotropic changes of the metal, — at least at higher temperatures, — occur.

Between 0° and about 800° C. the values of $\bar{c}_p$ in table II for pure, *oxygen-free* silver can, in function of the temperature, very well be expressed by the formula:

$$\bar{c}_p = 0{,}055401 + 0{,}7207 \, . \, 10^{-5} \, . \, t - 0{,}54053 \, . \, 10^{-9} \, . \, t^2 \quad . \quad . \quad \text{(I)},$$

and the corresponding *true* specific heats, therefore, by:

$$c_p = 0{,}055401 + 0{,}14414 \, . \, 10^{-4} \, . \, t - 0{,}16216 \, . \, 10^{-8} \, . \, t^2.$$

This formula can be applied between 0° and about 800° C.: the deviations of the values observed and calculated are all situated within the limits of experimental error. The atomic heats C_p can, therefore, be calculated, — within the same limits of the temperature, — by means of the formula:

$$C_p = 5{,}9767 + 0{,}1555 \, . \, 10^{-2} \, . \, t - 0{,}17494 \, . \, 10^{-6} \, . \, t^2.$$

Some values of C_p thus calculated are the following:

Temperature in °C.:	Atomic Heat C_p:
0°	5.977
100	6.130
200	6.281
300	6.427
400	6.571
500	6.710
600	6.847
700	6.979
800	(7.109)

This curve I for C_p is represented in Fig. 2; it may be considered as the *normal* curve for the atomic heats in their dependance on the temperature.

[1]) J. F. L. HAUSMANN, Abh. Kön. Ges. Wiss. Göttingen, (1850), 4; V. KOHLSCHÜTTER and E. EYDMANN, Ann. d. Chem., **390**, (1912), 340; **398**, (1913), 14; G. W. A. KAHLBAUM, K. ROTH and P. SIEDLER, Zeits. f. anorg. Chem., **29**, (1902), 177; W. HAIDINGER, Jahrb. Geol. R. Anst. Wien, **1**, (1850), 150; L. VEGARD a.o., conf. *Strukturbericht*, Erg. Bnd. Zeits. f. Kryst. (1931), 36—38, 748. The existence of a rhombic modification in electrolytically precipitated silver: G. ABLARD, Compt. rend. Paris, **187**, (1928), 223, is very dubious.

Just as formerly stated, this curve is *concave* towards the temperature-axis; the values of C_p are right within about $\pm$ 0.5 %. If, however, the abnormally steep change of $\bar{c}_p$ at higher temperatures shall also be taken into account, $\bar{c}_p$ can only be expressed pretty accurately by means of a formula with more than three constants, — for instance, by:

$$\left.\begin{aligned}\bar{c}_p = 0{,}05355 + 0{,}37946 \,.\, 10^{-4} \,.\, t - 0{,}15285 \,.\, 10^{-6} \,.\, t^2 + 0{,}3278 \,.\, 10^{-9} \,.\, t^3 - \\ - 0{,}32422 \,.\, 10^{-12} \,.\, t^4 + 0{,}1214 \,.\, 10^{-15} \,.\, t^5\end{aligned}\right\} \text{(II)}.$$

The true specific heat c_p is, therefore, expressed by:

$$c_p = 0{,}5355 + 0{,}75892 \,.\, 10^{-4} \,.\, t - 0{,}45855 \,.\, 10^{-6} \,.\, t^2 + 0{,}13112 \,.\, 10^{-8} \,.\, t^3 - \\ - 0{,}16211 \,.\, 10^{-11} \,.\, t^4 + 0{,}7284 \,.\, 10^{-15} \,.\, t^5,$$

and the atomic heat C_p by:

$$C_p = 5{,}7770 + 0{,}81873 \,.\, 10^{-2} \,.\, t - 0{,}49469 \,.\, 10^{-4} \,.\, t^2 + 0{,}14145 \,.\, 10^{-6} \,.\, t^3 - \\ - 0{,}17488 \,.\, 10^{-9} \,.\, t^4 + 0{,}7857 \,.\, 10^{-13} \,.\, t^5.$$

A series of values of c_p and C_p thus calculated are the following:

Temperature t in °C.	True specific Heats c_p :	Atomic Heats C_p :
350°	0.05966	6.436
400	0.06041	6.517
500	0.06220	6.710
600	0.06377	6.879
700	0.06491	7.003
800	0.06678	7.204
900	0.07276	7.849
950	0.07916	8.540

From the graph (Fig. 2) it becomes clear that, within the limits of experimental error, there is only above 780° C. a more considerable deviation between the two curves I and II: indeed, taking into account the exactness reached in the measurement of $\bar{c}_p$, the uncertainty in c_p is still about 0.5 %, so that both curves I and II may be considered as practically coïnciding up to about 800° C. Curve I evidently represents the true values of C_p in function of the temperature between 0° and 800° C.; its shape is the same as that previously found for stabilized silver.

The rapid increase of the curve II above about 780° C. is clearly manifested and certainly must be connected with the absorption of oxygen by the heated metal, as *it exactly begins at the temperature, at which,*

according to TROOST [1]), *the diffusion of oxygen through solid silver sets in with an observable velocity.*

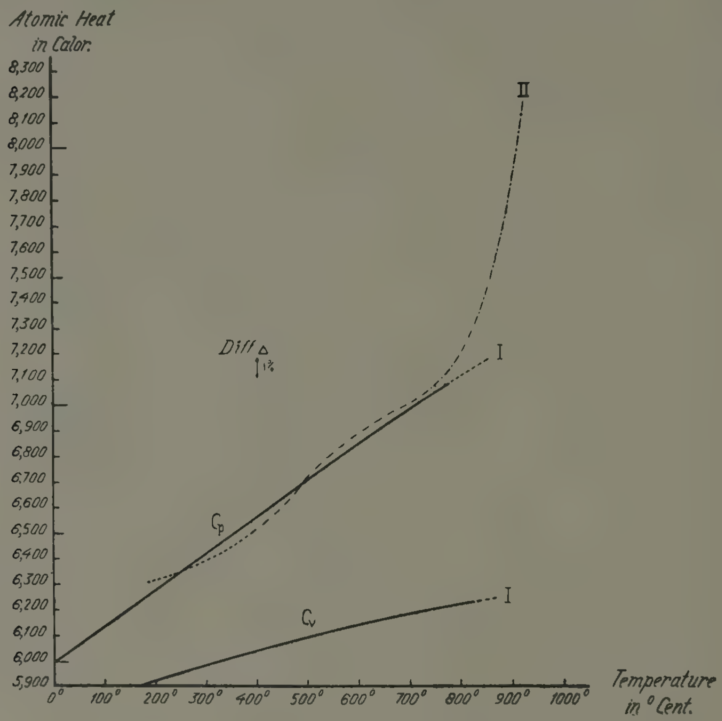


Fig. 2. True Atomic Heats C_p of Oxygenfree Silver.

Above 800° C., therefore, the values of $\bar{c}_p$ determined, evidently have no longer a real significance, as they depend on the accidental way of executing the experiments.

§ 3. The fact that *molten* silver absorbs considerable quantities of oxygen has already been known [2]) since 1819 and also the fact, that, on cooling, the metal gives off the total or the greater part of this oxygen by manifesting the phenomenon of "spitting". As to the question, whether *all* the gas absorbed by the liquid metal is really liberated on cooling and solidifying, there has been much controversy. DUMAS [3]), TROOST [4]) and other investigators stated, that the solidified metal, on heating, still gave off considerable quantities of oxygen ; but VAN DER PLAATS [5]), BRAUNER [6])

---

1) L. TROOST, loco cit.

2) S. LUCAS, Ann. de Chim. et Phys., (2), **12**, (1819), 402.

3) J. B. A. DUMAS, Compt. rend. Paris, **86**, (1878), 65; Ann. d. Chim. et Phys., (5), **14**, (1878), 289.

4) L. TROOST, Comp. rend. Paris, **98**, (1884), 1427.

5) J. VAN DER PLAATS, Rec. d. Trav. d. Chim. d. Pays-Bas, **5**, (1886), 212.

6) B. BRAUNER, Bull. Acad. de Belg., (3), **18**, (1889), 81.

and other investigators [1]) have contested this. HEYCOCK and NEVILLE [2]) found that the meltingpoint of silver in an atmosphere of oxygen is lowered by 10°—17°; CALLENDAR [3]) demonstrated, that molten and solidified silver, — after "spitting", — still showed a depression of its meltingpoint of 6°.4. LE CHATELIER [4]) found, that solid silver already at 300° C. takes up oxygen, if the pressure of the gas be greater than 12 atmospheres. According to MANCHOT [5]), silver is covered with a very thin film of its oxide, if it be heated in air at 200° C. TROOST [6]) demonstrated, that at 780° C. oxygen *diffuses* through solid silver, while to other gases, like CO, N_2, etc. it is impermeable under the same circumstances. As the concentration of the oxygen absorbed by the molten metal proves to be proportional to the square root of the oxygen-pressure [7]), the oxygen in it cannot be present as O_2, but must be present either *in the atomic state* [2]) or as Ag_2O. Between these two alternatives a final decision cannot be made (conf. DONNAN and SHAW [8])).

Although Ag_2O, once formed, is more or less rapidly decomposed already at 270°—300° C., all investigations hitherto made seem to indicate, that the affinity of oxygen for silver at higher temperatures is rather considerable [9]), but that the oxide formed then is decomposed at lower temperatures.

The experiments here described now clearly demonstrate, that the oxygen absorbed by the liquid metal is, on solidification, certainly *not completely* given off, but is partially occluded in the crystalline mass; moreover, that on preserving such an oxygen-containing lump at room-temperature for a long time, the gas is gradually set free by a very slow process of diffusion and that, reversely, by heating the solid metal in air at temperatures above about 650° C., oxygen is again absorbed. The slowness of its progress is readily explained by the extreme slowness of the diffusion of the gas through the crystalline mass of the metal at these temperatures. The method of measurement of the specific heats, however, appears to be sufficiently sensitive for revealing the absorption and the subsequent escaping of the gas, by the increase or by the diminution respectively of the values for $\bar{c}_p$ at each temperature.

---

1) L. MEYER and K. SEUBERT, Ber. d. d. chem. Ges., **18**, (1885), 1098.
2) C. F. HEYCOCK and F. H. NEVILLE, Journ. chem. Soc. London, **67**, (1896) 1024.
3) H. CALLENDAR, Phil. Mag., (5), **33**, (1892), 220.
4) H. LE CHATELIER, Bull. Soc. chim. Paris, (2), **48**, (1887), 342.
5) W. MANCHOT, Ber. d. d. chem. Ges., **42**, (1909), 3942.
6) E. TROOST, loco cit.
7) A. SIEVERTS and J. HAGENACKER, Zeits. f. Phys. Chem., **68**, (1910), 115.
8) F. G. DONNAN and T. W. A. SHAW, Journ. Soc. Chem. Ind., **29**, (1910), 987.
9) H. ST. CLAIRE DEVILLE and H. DEBRAY, Ann. de Chim. et Phys., (3), **56**, (1859), 385; **80**, (1875), 459; L. TROOST and P. HAUTEFEUILLE, Compt. rend. Paris, **84**, (1877), 946; V. KOHLSCHÜTTER and A. NOLL, Zeitschr. f. Elektrochem., **18**, (1912), 424; H. VON WARTENBERG, ibid., **19**, (1913), 489.

§ 4. From the data given in the literature concerning the compressibility[1]) and the linear expansion[2]) of silver, the approximate values of C_v can be calculated by means of the usual thermodynamical equation.

From the measurements, the linear expansion of silver between 0° and 875° C., can fairly well be expressed by the formula:

$$l_t = l_0 \; (1 + 0{,}000018033 \, . \, t + 0{,}000000004833 \, . \, t^2).$$

The following values of c_v and C_v thus are found:

Temperature in °C.	c_p :	c_v :	C_p :	C_v :
0°	0.05540	0.05329	5.977	5.748
100	0.05682	0.05415	6.130	5.841
200	0.05822	0.05495	6.281	5.928
300	0.05958	0.05557	6.427	5.995
400	0.06091	0.05610	6.571	6.052
500	0.06220	0.05652	6.710	6.098
600	0.06347	0.05701	6.847	6.150
700	0.06469	0.05743	6.979	6.195
800	0.06590	0.05775	(7.109)	6.230

These values of C_v are also graphically represented in Fig. 2; no indication is manifested of a maximum in this curve, as was found by MAGNUS and HODLER[3]).

Groningen, Laboratory for Inorganic and Physical Chemistry of the University.

---

1) P. W. BRIDGMAN, Proceed. Amer. Acad. Arts and Sciences, **58**, (1923), 165. For $\varkappa \, . \, 10^6$ at 30° C. the value: 0.987, at 75° C. the value: 1.004 was used and a linear increase of $\varkappa$ with the temperature was assumed.

2) L. HOLBORN and A. L. DAY, Ann. der Phys., (4), 2, (1900), 505; K. SCHEEL, Zeits. Phys., **5**, (1921), 167.

3) A. MAGNUS and A. HODLER, Zeits. f. phys. Chem., **110**, (1924), 188.

Anatomy. — *The anthropological features of Armenian and Assyrian (Aissori) women.* By C. U. ARIËNS KAPPERS.

(Communicated at the meeting of March 25, 1933).

Exact measurements concerning the women in the Near-East are rare. The anthropological statements usually concern men, a fact to be explained by the difficulty anthropologists meet in approaching the females.

This holds good especially for the Moslim population. But also for the Christian population of these regions anthropological data of females are few.

From this rule an exception is made by the work of Prof. and Mrs. KRISCHNER [1]), who contributed so much to our knowledge of the Near-East, including the females.

Among the papers left by the late Prof. H. KRISCHNER is a dossier containing the various somatic measurements of more than two hundred Armenian women.

The data registered for each are: age, height, sitting height, the breadth, length and heigth of the head, height and breadth of the nose, breadth and length of the ear, breadth and length of the face and the internal and external eye distances, all together the largest series of data ever measured on women of this race.

In addition Prof. KRISCHNER registered the same data for 37 Armenian males and, — which is important —, for fifteen female Assyrians (Aissori or Suriani). —

As Mrs. M. KRISCHNER put these data to my disposal I am giving the result of this work in the following pages.

Hitherto data concerning Armenian females are only given by CHANTRE [2]), who examined forty four Armenian women. KAPPERS gave us the average headmeasurements of 39 females, while the KRISCHNERS themselves reported the lengthbreadth indices of 43 North Persian Armenian women and 64 female Armenians from New-Julfa and Isphahan (l.c. supra).

Whether or not in TWARIANOWITSCH's dissertation [3]) or in ERIKSON's work [4]) data concerning women occur, I do not know as their work was not available to me.

---

[1]) The anthropology of Mesopotamia and Persia A.: Armenians, Khaldeans, Suriani (or Aissori) and Christian "Arabs" from Irak, B.: Jesidis and Moslim Arabs from Irak with some remarks on Kurds and Jews, C.: The anthropology of Persia. Proceedings of the Kon. Ak. v. Wetensch. Amsterdam. Vol. 35, 1932, p. 205—227 and p. 399—410.

[2]) CHANTRE. Recherches anthropologiques dans l'Asie occidentale. Mém. du Museum d'histoire naturelle de Lyon, Tome VI, 1895.

[3]) Materialien für die Anthropologie der Armenier. Dissertation of the military medical Academy of St. Petersburg. N⁰. 57, 1897.

[4]) Zur Anthropologie der Armenier. Russ. Anthrop. Journ. N⁰. 3—4, 1907.

In fig. 1 I give the curve of the Armenian women measured by Dr. H. KRISCHNER in Beirut.

This curve is an interesting one, first of all since it shows the usual Armenian peaks of 83 and 86—87, so constantly found especially with Armenian males. This again enhances the probability that these peaks have a racial and hereditary meaning.

It is, however, curious that, whereas in the curves of the Armenian women hitherto measured, as well by Prof. and Mrs. KRISCHNER themselves as by CHANTRE and KAPPERS, the subbrachycephalic (83) peak prevailed on the hyperbrachycephalic one (86—87), with the group of women measured at Beirut, both peaks are about the same, the latter being even slightly higher, as more commonly found with men.

This fact is the more striking, as in the Beirut curve, in addition to the hyperbrachycephalic 86—87 peak, there is an ultra-brachycephalic peak at 90.

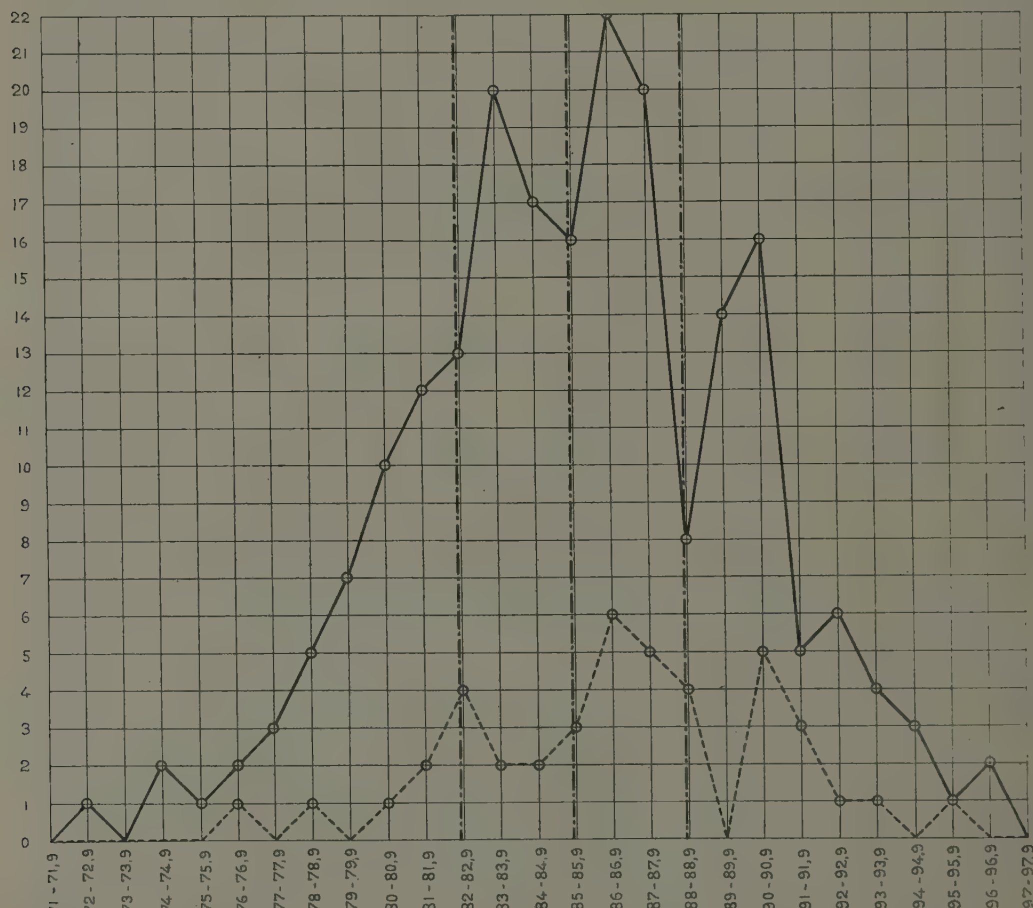


Fig. 1. Continuous line: cephalic index curve of 210 Armenian woman measured by Prof. H. KRISCHNER at Beirut. Dotted line: indexcurve of those women that only had 20 years of age or less.

The occurence of the latter peak is the more interesting as it is not present among the Armenian females examined by the KRISCHNERS in Persia, nor among the females of CHANTRE measured in the Caucasus chiefly.

The greater frequency of hyperbrachycephaly (86—87 index) as well as the frequent occurence of ultrabrachycephaly (90 index) among this material points to Anatolian influences. It is especially with the population of Anatolia (Asiatic Turkey) that hyperbrachycephaly prevails (with men as well as with women) as appears from VON LUSCHAN's [1]) and KAPPERS' [2]) Turkish curves, in which the 86—87 peak is by far the most outstanding one. Nevertheless even in these Turkish curves there are only very few ultrabrachycephalic (90) indices.

With another group of the Anatolian population, however, the 90 index was found to be very frequent, viz. with the „Greeks" measured by NEOPHYTOS [3]) at Kerasun on the South-Eastern coast of the Black Sea. Making a curve of the individual indices published by NEOPHYTOS (fig. 2) it appears that also here (in addition to the brachycephalic peak) a distinct ultrabrachycephalic (90) peak occurs.

Although the „Greeks" measured by NEOPHYTOS were men, this curve nevertheless shows that in the Eastern corner of Anatolia a good many ultrabrachycephalics occur.

That these individuals called themselves Greeks, proves little about their racial origin, since the Greek language and church have dominated so long in the Byzantine period, and also later, that there are certainly a great many original Anatolians among the "Greeks" of this region.

In connection with this fact it is interesting to note that the majority of the Armenians living at the present time in Beirut do not come from the Caucasus, Mesopotamia or Persia, but from Anatolia, whence they fled for the Turkish prosecutions. Whether their Anatolian origin or another factor [4]) (undersized growth? see below) is responsible for the large number of ultrabrachycephalics amongst them is difficult to say.

So much is sure these high indices do not agree with CHANTRE's theory concerning the influence of the cradle on the Armenian index. As mentioned elsewhere this author was inclined to explain the somewhat smaller average index of his Armenian females, compared with his Armenian males by supposing that the Armenian cradle [5]) favored brachycephaly and that

1) VON LUSCHAN. Völker, Rassen und Sprachen, Berlin 1922, p. 147.

2) KAPPERS. Contributions to the Anthropology of the Near-East N⁰. VI: Turks and Greeks. These Proceedings Vol. 34, 1931, p. 1088 and fig. 1.

3) NEOPHYTOS. Le Grèc du Nord-Est de l'Asie mineure. l'Anthropologie, Tome II, 1891, p. 25.

4) The age factor is not responsible for it as the women of 20 years and less do not show this more than the others (c.f. the dotted line in fig. 1).

5) In this cradle, of which I saw a specimen used in Aleppo, the child is more or less fixed, the head lying on the occiput.

the higher average cephalic index of the Armenian men were due to a more rigid application of this cradle with boys than with girls. This point

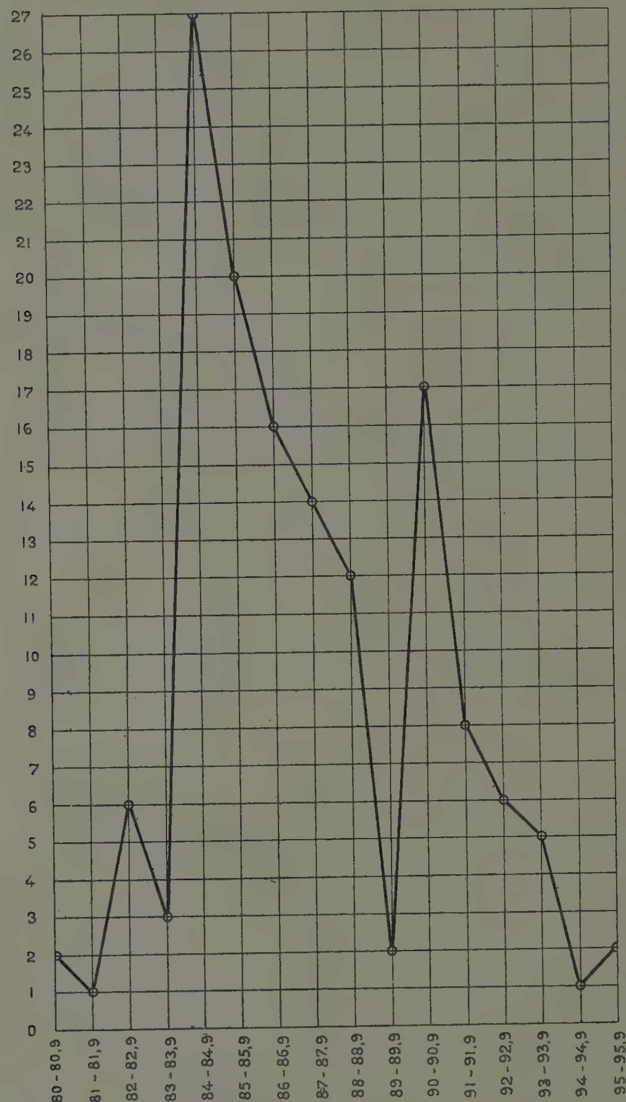


Fig. 2. Cephalic index of 142 "Greeks" measured by Neophytos at Kerasun in N. E. Anatolia.

of view, against which I advanced a number of arguments, now falls entirely, since it would be strange if in the Caucasus and North-Persia cradling were more rigidly practised with boys and in Anatolia with girls.

The natural explanation of the indicial differences is more likely to be sought for in other factors, especially in various racial admixtures.

In connection with this possibility I also refer to DJAWACHISCHWILI's in-

teresting paper[1]) where he says: „es ist möglich, dass es Unterschiede zwischen den Armeniern Armeniens, Georgiens und der Turkei gibt, dass ferner die Armenische Gruppe mehrere Typen enthält."

That the ultrabrachycephalic element forms a special group seems to be confirmed by the other indices measured by Dr. KRISCHNER with these women, the nasal index, the anatomical facial index, the transverse cephalo-facial index, stature and sitting height.

The nasal index shows great individual variations. Calculating the average index of each of the four groups of cephalic indices separated by the vertical lines in figure 1, it appears that the average nasal index tends to increase with the larger cephalic index group. With the 81 and lower cephalic index group the average nasal index is *58.3,* with the 82, 83, 84 cephalic index group it is *62.4,* with the 85, 86, 87 group it is 64.4 and with the 88 and higher cephalic index group it is *65.2.* —

Considering the two middle groups as typical Armenian, the average female Armenian nasal index would be *63.4* (the average for 43 females examined by CHANTRE was 66.0). —

With the anatomical facial index it appears that individual variations in each of the groups are greater than with the nasal index.

The averages, however, approach each other more than with the nasal index. The average anatomical facial index of individuals with a cephalic index of 81 and less is 87,8 ; for the 82,83 and 84 group it is *87,2* ; and for the 85, 86, 87 peak *86,6,* while for the cephalic indices of 88 and higher it is *85,9.* The variations in the anatomical facial index in each group, however, are too great to attach much importance to these average figures. Taking again the two middle groups as typical Armenian this would give an average female facial index of 86,9 [2]).

There is some more correlation between the length-breadth index of the head and the transverse cephalo-facial index.

For the 81 and lower cephalic index group the average transverse cephalo-facial index is *91,* with the 82, 83, 84 cephalic index group it is *88.3,* with the 85, 86, 87 group it is *87.7* and with the 88 and higher index group it is *86.5.* Taking again the two middle groups together this would give an average female transverse cephalo-facial index of *88.0.* —

Comparing cephalic index and body height and leaving out the cephalic index group of 80 and less (as these indices are certainly not characteristic of Armenians), calculating only the (81), 82, 83, 84, the 85, 86, 87 and the 88 and higher index groups, it appears that the two first named groups have about the same average height (153 cm. $\pm$ 0.419) [3]), while the group with indices of 88 and higher are slightly smaller people (151.9 cm.).

---

[1]) DJAWACHISCHWILI. Die Rassenzusammensetzung der Kaukasusvölker. Archiv. f. Anthropologie. Bnd. XX 1925, p. 77 (especially p. 801).

[2]) The average facial index found by CHANTRE in his Armenian females differs from this (103,1), but this author measured the index in a different way.

[3]) CHANTRE found exactly the same average height for his females.

This is more obvious if I calculate the averages for some indices only. For the 81 cephalic index the average stature is *155.3*, for the 83—84 group *152.9*, for the 86—87 group *152.65* and for the 90 index only *151.1* cm With the indices of 93 and higher the average bodyheight is even somewhat under 150 cm.

The same holds good for the sitting height. For the 83—87 index group the average sitting height is 82.2 cm., but for the 90 cephalic index peak it is only 81.2 cm. Although this agrees with the results obtained by PITTARD and DONICI [1]), the correlation coefficient, calculated according to the formula of BRAVAIS is too small (0.06) to speak of a "law" in this case.

The fact that with the groups 81—84 and 85—88, representing the specific Armenian peaks, the average body and sitting height are nearly the same, is in favour of the closer relationship between these two groups.

I finally mention the average earlength: 5.99 cm. in these Armenian women (CHANTRE's women 5.3 cm.). In the men registered by Prof. KRISCHNER it is 6.71 cm.

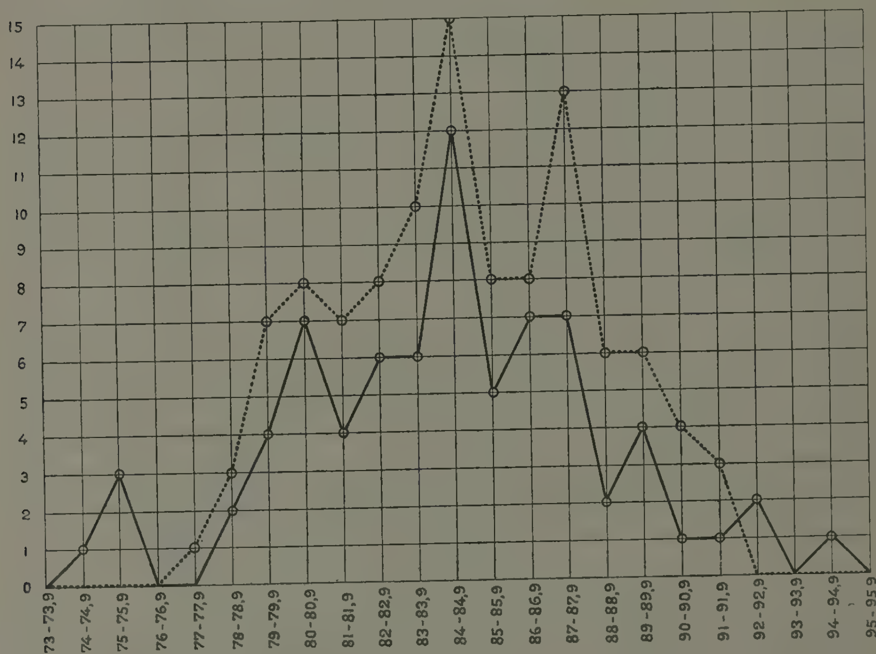


Fig. 3. Continuous line = cephalic index curve of 75 Aissori women. Dotted line the same + the indices of 34 Aissori males.

In the same dossier left by Prof. KRISCHNER there are complete cephalic and somatic data concerning fifteen Assyrian women. Although this number is small, their cephalic indices together with the cephalic indices of

[1]) PITTARD. l'Influence de la taille sur l'indice céphalique dans une groupe ethnique relativement pur. Bull. Soc. d'Anthropologie de Paris 1905 and PITTARD et DONICI: Les changements de l'indice céphalique en fonction de la taille croissante. Ibidem 1927.

Aissori women published before by Prof. and Mrs. KRISCHNER give us a total of 58 female Assyrian indices.

In fig. 3 I reproduce these indices, adding a few female Aissori indices measured by CHANTRE and KAPPERS. From this it appears that the Aissori curve shows a paralellism to the curve of the female Armenian indices if we leave out of consideration the ultra-brachycephalics female Armenians discussed above. This again proves the close relation existing between these ethnic groups, a fact also emphasised bij DJAWACHISCHWILI (l.c. supra) and the KRISCHNERS, and confirmed by PARR's [1]) researches on the blood index of these groups (Armenians 2.31, Assyrians 2.19).

The average body height of the 15 Aissori women, mentioned by Prof. KRISCHNER in 154.1 cm. (i.e. 1.1 cm. higher than with the Armenian women), the average cephalic index is 84.8, the average transverse cephalo-facial index is 90.3, the anatomical facial index 85.6, the nasal index 65.2, sitting height 82.1 cm., earlength 6.09 cm.

The joint table shows all the averages found:

Group	L. br. index	S. H.	Bodyh.	Transv. c. f. i.	Anat. fac. i.	Nasal. i.	Ear length
Armen. ♀	85.2 ± 0.36	82.2	153 cm.	88.0	86.9	63.4	5.99 cm.
Assyr. ♀	84.8	82.1	154.1 cm.	90.3	85.6	65.2	6.09 cm.

Resuming our results concerning the Armenian women measured at Beirut by Dr. KRISCHNER we may say that with these women the sub-brachycephalic as well as the hyperbrachycephalic indices, so characteristic of Armenian males, occur. In addition, an index of 90 is very frequent. The correlation with the other somatic features makes it probable that the ultrabrachycephalic 90 index people is a special group. Excepting the ultra-brachycephalic indices analogous relations are found [2]) with the Assyrian women measured by the same author. The same resemblance existing between the Armenian and Assyrian males, the relations with both sexes point to a common origin of the Armenians and Aissori.

The stock from which both peoples originate is the Subarean or Ponto-zagrian stock (E. SPEISER [2]), to which the proto-Armenians (Hittites) and probably the majority of the ancient Assyrians belong.

Of the anthropological status of the Hittites and ancient Assyrians little is known since hitherto no skulls of these peoples are found with certainty.

---

1) PARR. Die Blutgruppenverteilung in der Bevölkerung des nahen Ostens und Nord-Afrika. Ukrainisches Zentralblatt für Blutgruppenforschung, Bnd. 4, 1930, und PARR Blood studies on people in Western Asia and North Africa. Amer. Journ. of Physical Anthropology, Vol. 16, 1931.

2) E. SPEISER. Mesopotomian Origins. University of Pennsylvania Press, 1930.

The only way to know something about their somatic, especially cephalic features is to study ancient sculptures and mural engravings.

As far as concerns the Hittites I have already pointed out in these Proceedings Vol. 33, 1930, the resemblance between the Armenian males and Hittite warriors captured by Ramses the Great and represented at Medinet Habu. It is much more difficult to find representations of Hittite and Assyrian women. In the adjoining plates I give what I have been able to find, adding some Armenian women and men for comparison.

This plate clearly shows that all the people represented here is characteristic by the postauricular shortness of the head, the strong, frequently curved nose, pronounced chin, large eyes and ears and strong lips. In the most ancient Hittite pictures (second row) some of these features are even considerably charged, thus producing the wellknown bird head type, so characteristic of this ethnic group, as already emphasized by HÜSING [1]) and OPPENHEIM [2]).

In a former paper [3]) I stated that, according to my opinion the Armenians and Assyrians are not the only representatives of this stock, but that the Northern or Ashkenazim Jews (c.f. also VON LUSCHAN) and the Kohtan Arabs (c.f. also SELIGMAN) may also be related to it, especially to the subbrachycephalic component of this group (the hyperbrachycephalic component being more frequent among the Anatolians). —

As shown before the Sephardim or Southern Jews represent much more a Canaanite-Phoenician (= Asiatic mediterranean) type. As the Sephardim Jews are supposed to be related to the Southern tribes (Judah and Benjamin) this result would agree with Prof. ROBINSON's [4]) conception that the tribe of Judah was strongly mixed with Canaanites. This also appears from the fact that the semitic skulls which MACALISTER [5]) found at Gezer in Judea, and dating from about 2500—550 B.C. are mesocephalic, mixed with dolichocephalics of probably Egyptian origin. The outstanding 77 peak of the Gezer cranial index curve is distinctly Phoenician i.e. Canaanite. Similarly of the six skulls of the second and first century B.C. found near Jerusalem, four of which by BRANDENBURG [6]) (described by H. VIRCHOW [7])), and two by HENCKEL [8]), three are mesocephalic,

---

1) HÜSING. Völkerschichten in Iran. Mitteil. der Anthrop. Gesellsch. in Wien. Bnd. 44, 1916.

2) VON OPPENHEIM. Der Tell Halaf. Brockhaus, Leipzig, 1931.

3) These Proceed. Vol. 34, 1931.

4) ROBINSON. The origin of the tribe of Judah. Amicitiae Corolla. A volume of Essays presented to Dr. J. RENDEL HARRIS. Univ. of London. Press 1933, see also: The same: Actes du XVIII Congrès international des orientalistes, Brill, Leyden, 1932.

5) MACALISTER. The excavation of Gezer, Vol. I, p. 60, published for the committee of the Palestine exploration fund, London, 1912.

6) BRANDENBURG's report of his findings is given in the Zeitschr. f. Ethnologie, 1924.

7) H. VIRCHOW's description of BRANDENBURG's skulls is found in the Zeitschr. f. Ethnologie Jhrg. 60, 1928, p. 154.

8) For the description of two skulls found by HENCKEL see Zeitschr. für Morphologie und Anthropologie 1930, p. 236.

C. U. ARIËNS KAPPERS: THE ANTHROPOLOGICAL FEATURES OF ARMENIAN AND ASSYRIAN (AISSORI) WOMEN.

two dolichocephalic, only one being subbrachycephalic. Though also with the present Samaritans the Phoenician-Canaanite type is still in the majority, the number of subbrachycephalics amongst them has considerably increased. Since these subbrachycephalics prevail now-a-days in North Palestine and in the North-Western desert border cities this seems to prove a secundary influx from the North East after the Phoenician-Canaanite migration. Whether this secondary influx is to be connected with the Chabiru or (later) Aramean invasion, or both, cannot be answered with certainty at the moment [1]).

EXPLANATION OF PLATE:

Top row. From left to right: Armenian woman, Armenian man, Hittite warriors, from an engraving at Medinet Habu, about 1350 B.C. (after COWLEY's Schweich lecture on the Hittites, Oxford University Press, 1920).

Second row. Left: two Hittite women, from an engraving on a tomb stele from the middle of the second millen. B.C. near Marash (from ED. MEYER. Reich und Kultur der Chehiter, Berlin 1914), right: Hittite woman on a tomb stele near Senjerli (after GARSTANG: The Hittite empire, London 1929) dated about the middle of the second millennium.

Third row. An Assyrian woman, British museum (from WELL's Outline of History) between an Armenian male and female.

Fourth row. An Assyrian woman (8th cent. B. C. LOUVRE, Paris) between the front and profile portrait of an Armenian woman.

[1]) Prof. F. M. TH. BÖHL, Leyden, writes me that according to his linguistic-archeological studies the Arameans (and probably their Chabiru precursors), coming from the North-East via the Damascene region have wedged into Palestine between the Northern (now Syrian) district and Southern Palestine. This would be in perfect harmony with my anthropological results. See also BÖHL Kanaanäer und Hebräer, Leipzig 1911, p. 14 and 26.

Mathematics. — *Ueber eine in der Theorie der Funktionen von zwei komplexen Veränderlichen auftretende unitäre Geometrie.* Von STEFAN BERGMANN. (Communicated by Prof. J. G. VAN DER CORPUT).

(Communicated at the meeting of March 25, 1933).

In der Theorie der komplexen Orthogonalfunktionen treten gewisse Grössen auf, die bei einer analytischen Transformation des Definitionsgebietes in ein anderes sich wie kovariante Hermitesche Tensoren transformieren. Sie definieren also eine unitäre Geometrie.

Herr SCHOUTEN und Herr VAN DANTZIG haben in einigen bekannten Arbeiten die unitären Geometrien in differentialgeometrischer Hinsicht

untersucht. In der vorliegenden Note betrachten wir die in der Funktionentheorie auftretende unitäre Geometrie und zeigen allgemein folgendes: Jede unitäre Geometrie, bei der der Fundamentaltensor $g_{\nu\bar{\Omega}}$ sich als kovariante Ableitung des Logorithmus einer sich wie die Dichte transformierenden Grösse Φ darstellen lässt, ist symmetrisch, ferner besteht zwischen dem Krümmungs-, Fundamentaltensor $g_{\nu\bar{\Omega}}$ und den kovarianten Ableitungen eines Skalars stets eine lineare Beziehung. (Im Falle der in der Funktionentheorie auftretenden unitären Geometrie wurde diese Beziehung bereits in der Arbeit *h.* (s. u.) angegeben. In diesem Falle hat Φ eine wichtige funktionentheoretische Bedeutung, es ist die sogenannte "Kernfunktion" des Bereiches.)

In Zusammenhange mit diesen Betrachtungen werden die Invarianten gegenüber konformer Abänderung des Fundamentaltensors aufgestellt und die notwendigen und hinreichenden Bedingungen angegeben, wann eine unitäre Geometrie konformeuklidisch ist.

§ 1. In der Arbeit *a.* hat Herr SCHOUTEN [1]) die differentialgeometrischen Methoden auf die Hermiteschen Grössen ausgedehnt und u.a. gezeigt, dass ein Hermitescher Tensor n-ten Ranges eine lineare Übertragung erzeugt. Verlangt man, dass die Übertragung gewissen natürlichen Bedingungen genügt, so ist, wie es Herr SCHOUTEN zeigt, durch die Forderung, dass das kovariante Differential verschwindet, die unitäre Geometrie eindeutig festgelegt.

Wendet man auf ein (vierdimensionales) Gebiet $\mathfrak{B}$ des (komplexen) $z^1 z^2$-Raumes eine analytische Transformation

$$z^{*k} = z^{*k}(z^1, z^2), \qquad k = 1, 2 \quad \ldots\ldots \quad (1.1)$$

an [2]), so kann man dies auch so auffassen, dass das Gebiet unverändert bleibt, und wir von den Koordinaten (z^1, z^2) zu den neuen Koordinaten (z^{*1}, z^{*2}) übergehen.

Es liegt nahe, auch in der Funktionentheorie bei den Fragen, die mit der Abbildung von Gebieten zusammenhängen, die differentialgeometrischen Methoden anzuwenden. Es muss dabei jedoch folgendes berücksichtigt werden:

I. In der Differentialgeometrie betrachtet man alle Grössen im Kleinen. In der Funktionentheorie ist es bei vielen Problemen gerade von Wichtig-

---

1) *a.* SCHOUTEN, "Ueber unitäre Geometrie", Proc. Kon. Akad. v. Wet. Amsterdam **32** (1929) S. 457—465. Vgl. auch die Arbeiten: *b.* SCHOUTEN und VAN DANTZIG, "Ueber die Differentialgeometrie einer Hermiteschen Differentialform ....", Proc. Kon. Akad. v. Wet. Amsterdam **32** (1929) S. 61—64; *c.* SCHOUTEN und VAN DANTZIG, "Ueber unitäre Geometrie", Math. Annalen **103** (1930) S. 319—346; *d.* SCHOUTEN und VAN DANTZIG, "Ueber unitäre Geometrie konstanter Krümmung", Proc. Akad. v. Wet. Amsterdam **34** (1931) S. 1293—1304.

Wir setzen im folgenden die Kenntnis der Arbeiten *a.* und *c.* voraus und übernehmen die SCHOUTENschen Definitionen und Bezeichnungen.

2) Die folgenden Betrachtungen lassen sich ohne jede Beschränkung oder Schwierigkeit auf den Fall der n komplexen Veränderlichen ausdehnen.

keit, dass man die zu untersuchenden Funktionen im ganzen Gebiete d.h. im Grossen betrachtet [3]). Die unitäre Geometrie muss also bei Untersuchung der funktionentheoretischen Fragen im Grossen definiert werden.

II. Eines der wichtigen Probleme der Funktionentheorie besteht darin, zu entscheiden, ob sich zwei gegebene Gebiete analytisch auf einander abbilden lassen bzw. die Abbildungsfunktion zu berechnen. Deswegen ist es wichtig, dass man die Möglichkeit hat, aus der Gestalt des Gebietes den Fundamentaltensor $g_{\nu\Omega}$ effektiv zu berechnen, ohne die Abbildungsfunktionen des Gebietes auf gewisse Normalbereiche zu kennen.

In der Theorie der komplexen Orthogonalfunktionen [4]) tritt eine wichtige Grösse auf, nämlich der Kern des im Bereiche vollständigen Orthogonalfunktionensystems, Kernfunktion des Bereiches. Wie in den Arbeiten *g.* und *h.* gezeigt ist, kann man mit Hilfe der Kernfunktion eine gegenüber analytischen Transformationen invariante Hermitesche Differentialform angeben, die also eine unitäre Geometrie bestimmt. Diese Geometrie erfüllt die in I und II formulierten Forderungen (vgl. dazu Arbeit *h* § 1).

Da die Kenntnis der in der Fussnote [4]) zitierten Arbeiten hier nicht vorausgesetzt wird, wollen wir an eine (von dem Begriff der Orthogonalfunktionen unabhängige) Definition der Kernfunktion und an einige ihrer Eigenschaften errinern [5]).

Sei $\mathfrak{B}$ ein schlichter, offener einfach zusammenhängender Bereich des $z^1 z^2$-Raumes und (t^1, t^2) ein innerer Punkt von $\mathfrak{B}$. Zieht man zur Konkurrenz alle in $\mathfrak{B}$ regulären Funktionen heran, die der Nebenbedingung

$$h(t^1, t^2) = 1 \qquad \ldots\ldots\ldots\ldots \quad (1.2)$$

---

[3]) Bei vielen Problemen hat man natürlich die Betrachtungen im Kleinen mit Erfolg angewandt. Vgl. insbesondere WIRTINGER "Zur formalen Theorie der Funktionen von mehreren komplexen Veränderlichen", Math. Annalen **97** (1927) S. 357—375.

[4]) Vgl. dazu *e.* "Ueber Hermitesche Formen, die zu einem Bereiche gehören", Sitz.-Ber. d. Berl. Math. Ges. **26** (1927) S. 178—184; *f.* "Ueber Hermitesche unendliche Formen, ...", Math. Zeitsch. **29** (1929) S. 641—677; *g.* "Zur Funktionentheorie zweier kompl. Ver." Jahresbericht d. Deutsch. Math.-Ver. **41** (1932) S. 78—80; *h.* "Ueber die Kernfunktion eines Bereiches und ihr Verhalten am Rande. I.", Crelles Journal für reine und angewandte Math. **169** (1933) S. 1—42; *i.* "Zur Funktionentheorie zweier komplexen Veränderlichen", Verh. d. intern. Kongresses in Zürich (1932) S. 59—61.

In der Arbeit *h.* S. 5, Zeile 12 von oben muss es

$$N = \frac{1}{6K_{\mathfrak{B}}^3} \sum_{\varkappa=1}^{\infty} \sum_{\mu=1}^{\infty} \sum_{\nu=1}^{\infty} |D_{\varkappa\mu\nu}(z_1, z_2)|^2$$

und nicht $N = \frac{1}{6} \sum_{\varkappa=1}^{\infty} \sum_{\mu=1}^{\infty} \sum_{\nu=1}^{\infty} |D_{\varkappa\mu\nu}(z_1, z_2)|^2$ heissen.

[5]) Der Nachweis der Aequivalenz der verschiedenen Definitionen der Kernfunktion und die Beweise der hier referierten Behauptungen befinden sich in den Arbeiten *f.* § 1 und *h.* § 1.

genügen so hat, wie in der Arbeit *f.* gezeigt wurde, das Integral

$$\int_{\mathfrak{B}} |h(z^1, z^2)|^2 \, d\omega \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1.3)$$

($d\omega$ — vierdimensionales Volumenelement)

ein Minimum. Der reziproke Wert dieses Minimums definiert in jedem Punkte (t^1, t^2) eine reelle Funktion, und zwar die Kernfunktion $K_{\mathfrak{B}}(t^1, t^2; \bar{t}^1, \bar{t}^2)$ des Bereiches $\mathfrak{B}$. Die Kernfunktion ist eine in $\mathfrak{B}$ reguläre Funktion [6]. Vermittelt (1.1) eine schlichte und eineindeutige Abbildung des Gebietes $\mathfrak{B}$ auf $\mathfrak{B}^\star$, so gilt für die Kernfunktionen der beiden Gebiete [7] die Relation

$$\left.\begin{aligned} K_{\mathfrak{B}^\star}(z^{\star 1}, z^{\star 2}; \bar{z}^{\star 1}, \bar{z}^{\star 2}) &= K_{\mathfrak{B}}(z^1, z^2; \bar{z}^1, \bar{z}^2)\, E(z^{\star 1}, z^{\star 2})\, \bar{E}(\bar{z}^{\star 1}, \bar{z}^{\star 2}) \\ E(z^{\star 1}, z^{\star 2}) &= \frac{\partial(z^1, z^2)}{\partial(z^{\star 1}, z^{\star 2})}, \quad z^k = z^k(z^{\star 1}, z^{\star 2}), \quad k = 1, 2 \end{aligned}\right\} . \quad (1.4)$$

Die analytischen bzw. antianalytischen Funktionen befriedigen die CAUCHY-RIEMANNschen Differentialgleichungen $\partial_\Omega \varphi = 0$ (bzw. $\partial_\omega \varphi = 0$). Es folgt daraus dass für eine reguläre Funktion $p(z^1, z^2, \bar{z}^1, \bar{z}^2)$, die sich als Summe einer analytischen und einer antianalytischen Funktion

$$p(z^1, z^2, \bar{z}^1, \bar{z}^2) = \varphi(z^1, z^2) + \psi(\bar{z}^1, \bar{z}^2) \quad . \ . \ . \ . \quad (1.5)$$

schreiben lässt, die Beziehung

$$\partial_\nu \partial_\Omega \, p = 0 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1.6)$$

gilt [8]. Da nach (1.4): $\log K_{\mathfrak{B}^\star} = \log K_{\mathfrak{B}} + [\log E(z^{\star 1}, z^{\star 2}) + \log \bar{E}(\bar{z}^{\star 1}, \bar{z}^{\star 2})]$ ist, gewinnt man nach (1.6) in

$$g_{\nu\Omega} = \partial_\nu \partial_\Omega \log K_{\mathfrak{B}} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (1.7)$$

einen kovarianten Hermiteschen Tensor, der in jedem Punkte von $\mathfrak{B}$ also im Grossen erklärt ist [9].

6) Unter einer regulären Funktion $f(z^1, z^2, t^1, t^2)$ verstehen wir eine Funktion der vier komplexen Veränderlichen z^1, z^2, t^1, t^2, die sich in der Umgebung jedes Regularitätspunktes in einem genügend kleinen (achtdimensionalen) Polyzylinder $|z^k| \leqq \sigma$, $|t^k| \leqq \sigma$, $k = 1, 2$, in eine gleichmässig konvergente Reihe entwickeln lässt. Die Kernfunktion ist in der Terminologie von SCHOUTEN eine halbanalytische Funktion (vgl. Arbeit *c.* § 2).

7) Vgl. insbesondere Arbeit *h.* Formel (1.15).

8) Es ist auch offenbar das umgekehrte richtig: ist p regulär und gilt für p (1.6), so folgt daraus die Darstellung (1.5); denn man kann dann p in eine gleichmässig konvergente Reihe entwickeln und die formel durch die Differentiation daraus gebildete Reihe stellt $\partial_\Omega \partial_\nu p$ dar. Da bei der Differentiation nur die Koeffizienten der nur von z^1, z^2 oder nur von $\bar{z}^1, \bar{z}^2$ abhängigen Glieder verschwinden, folgt aus (1.6), dass alle übrigen gleich 0 sind, und also p sich in der Form (1.5) darstellen lässt. Vgl. dazu ebenfalls die in der Fussnote 3) zitierte Arbeit von WIRTINGER S. 358.

9) Für das Linienelement ds der unitären Geometrie wurde in den Arbeiten *g.* und *h.*

$$\sqrt{I_{\mathfrak{B}} \partial_\nu \partial_\Omega \log K_{\mathfrak{B}} dz^\nu d\bar{z}^\Omega} \quad \text{bzw.} \quad \sqrt{\partial_\nu \partial_\Omega \log K_{\mathfrak{B}} dz^\nu d\bar{z}^\Omega}$$

genommen. $I_{\mathfrak{B}}$ bedeutet dabei eine gegenüber Koordinatentransformation invariante Grösse (Skalar).

Schliesslich sei hervorgehoben, dass für eine grosse Klasse von einfachzusammenhängenden Gebieten der Forderung II entsprechend eine effektive Berechnung der Kernfunktion, und somit von $g_{\nu\Omega}$ möglich ist.

§ 2. Satz. *Für die Bestimmungsgrössen eines Hermiteschen kovarianten Fundamentaltensors $g_{\nu\Omega}$ gelte:*

A. *Es gibt eine in $z^1, z^2, \bar{z}^1, \bar{z}^2$ reguläre Funktion $T(z^1, z^2, \bar{z}^1, \bar{z}^2)$ derart, dass*

$$g_{\nu\Omega} = \partial_\nu \, \partial_\Omega T, \quad \ldots \ldots \ldots \quad (2.1)$$

ist;

B. *es gibt eine analytische Funktion $\varphi(z^1, z^2)$ und eine antianalytische Funktion $\psi(\bar{z}^1, \bar{z}^2)$ $[\varphi \neq 0, \psi \neq 0]$, so dass $e^T \varphi \psi$ sich wie eine Dichte vom Gewicht $(+1, +1)$ transformiert.*

Die zu $g_{\nu\Omega}$ zugehörige lineare Uebertragung hat dann die folgenden Eigenschaften:

I. *sie ist symmetrisch, d. h., es gilt*

$$S_{\dot\lambda\dot\mu}^{\cdot\cdot\nu} = 0 \quad \ldots \ldots \ldots \ldots \quad (2.2)$$

(woraus $R_{\dot\Omega\dot\nu\dot\lambda}^{\cdot\cdot\cdot\nu} = R_{\dot\Omega\dot\lambda\dot\nu}^{\cdot\cdot\cdot\nu}$ folgt);

II. *es gibt einen Skalar* I, *so dass die Beziehung*

$$g_{\lambda\Omega} + R_{\dot\Omega\dot\lambda\dot\nu}^{\cdot\cdot\cdot\nu} = \partial_\lambda \, \partial_\Omega \log \mathrm{I} \quad \ldots \ldots \quad (2.3)$$

gilt.

Hat umgekehrt die Uebertragung die Eigenschaften I *und* II, *so gibt es eine Funktion T, so dass für T sowohl* A. *wie* B. *gilt.*

Beweis. Die Tatsache, dass die Voraussetzung A. die Eigenschaft I der Uebertragung nach sich zieht und vice versa folgt aus den Betrachtungen des § 5 der unter c. zitierten Arbeit von SCHOUTEN und VAN DANTZIG. (Vgl. insbesondere [46], [47] und [53] [10].)

Wir wollen nun zeigen, dass aus B. die Eigenschaft II folgt. Nach [56] gilt für jede unitäre Geometrie

$$R_{\dot\Omega\dot\mu\dot\alpha}^{\cdot\cdot\cdot\alpha} = -\partial_\mu \, \partial_\Omega \log \mathfrak{g}, \qquad \mathfrak{g} = |g_{\nu N}| \text{ [11]}) . \quad \ldots \quad (2.4)$$

Nun transformiert sich $\mathfrak{g}$ wie eine Dichte vom Gewicht $(+1, +1)$. Daraus folgt in Verbindung mit B. weiter, dass $\dfrac{e^T \varphi\psi}{\mathfrak{g}} = \mathrm{I}$ ein Skalar ist. Ersetzt man in (2.4) $\log \mathfrak{g}$ durch $T + \log \varphi + \log \psi - \log \mathrm{I}$ so gelangt man nach (1.5), (1.6) und (2.1) zu (2.3).

---

[10]) Durch eckige Klammern werden die Formeln der unter c. genannten Arbeit gekennzeichnet.

[11]) In der Formel [56] ist ein Druckfehler: Es soll $\mathfrak{g} = |g_{\nu N}|$ und nicht $\mathfrak{g} = |g^{\nu N}|$ heissen.

Da umgekehrt nach [56] für jede Übertragung die Beziehung (2.4) gilt, so folgt aus (2.3), dass das Bestehen von II die Beziehung

$$\partial_\lambda \partial_\Omega \log \mathrm{I} = g_{\lambda\Omega} - \partial_\lambda \partial_\Omega \log \mathfrak{g} = \partial_\lambda \partial_\Omega T - \partial_\lambda \partial_\Omega \log \mathfrak{g} \quad . \quad . \quad (2.5)$$

nach sich zieht. Da $\mathfrak{g}$ regulär ist und nicht verschwindet, folgt (vgl. Fussnote [8]) S. 310), dass I die Gestalt

$$\mathrm{I} = \frac{e^T \varphi \psi}{\mathfrak{g}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.6)$$

hat. $e^T \varphi \psi$ transformiert sich also wie eine Dichte vom Gewicht $(+1, +1)$.

Aus dem hier bewiesenen Satz ersehen wir, dass die aus der Kernfunktion gewonnene unitäre Geometrie nicht auf den allgemeinsten von Herrn SCHOUTEN betrachteten Fall führt. Es drängt sich sofort die Frage auf, ob man die in der Arbeit *g.* angegebene Methode unitäre Geometrien zu bilden nicht dadurch verallgemeinern kann, dass man auch spezielle Klassen von Funktionen bei der Minimumaufgabe zur Konkurrenz zulässt. Gibt es dann zu jeder unitären Geometrie mit den Eigenschaften I und II, die in einem Bereiche $\mathfrak{B}$ definiert ist, eine Klasse von in $\mathfrak{B}$ regulären Funktionen deren Kernfunktion die vorgegebene Geometrie erzeugt? Auf diese Frage, die das Heranziehen gewisser Sätze aus der Theorie der Orthogonalfunktionen erfordert, soll an einer andere Stelle eingegangen werden.

§ 3. Neben (1.7) kann man der Kernfunction $K_{\mathfrak{B}}$ ebenfalls den Tensor $'g_{\nu\Omega} = \mathrm{I}_{\mathfrak{B}}\, \partial_\nu \partial_\Omega \log K_{\mathfrak{B}}$ zuordnen, wo $\mathrm{I}_{\mathfrak{B}}$ eine beliebige invariante (Skalar) bedeutet. Im Zusammenhange damit entsteht nun die Frage nach den Invarianten bei konformer Abänderung.

Seien $g_{\nu\Omega}$ bzw.

$$'g_{\nu\Omega} = \sigma g_{\nu\Omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.1)$$

zwei kovariante Hermitesche Fundamentaltensoren, die durch eine konforme Abänderung auseinandern hervorgehen. Aus (3.1) folgen nun die Beziehungen

$$'\Gamma^{\nu}_{\mu\lambda} = {}'g^{\nu\Omega}\, \partial'_\lambda\, g_{\mu\Omega} = g^{\nu\Omega}\, \partial_\lambda\, g_{\mu\Omega} + g^{\nu\Omega}\, g_{\mu\Omega} \frac{\partial_\lambda \sigma}{\sigma} = \Gamma^{\nu}_{\mu\lambda} + A^{\nu}_{\mu}\, \partial_\lambda \log \sigma \,. \quad (3.2)$$

$$'R_{\dot\Omega\dot\mu\lambda}{}^{\nu} = -\partial_\Omega\, '\Gamma^{\nu}_{\lambda\mu} = -\partial_\Omega(\Gamma^{\nu}_{\lambda\mu} + A^{\nu}_{\lambda}\, \partial_\mu \log \sigma) = R_{\dot\Omega\dot\mu\lambda}{}^{\nu} - A^{\nu}_{\lambda}\, \partial_\mu \partial_\Omega \log \sigma \quad (3.3)$$

Aus (3.3) ersehen wir dass eine ganze Klasse von Abänderungen, nämlich diejenigen wo

$$\sigma = |\, F(z^1, z^2)\,|^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.4)$$

ist ($F \neq 0$), eine besondere Stellung in der unitären Geometrie einnimmt: bei einer solchen Abänderung des Fundamentaltensors bleibt nach (1.5), (1.6) die Krümmungsgrösse nämlich invariant.

Führen wir die Tensoren $F_{\mu\Omega}$ und $R_{\mu\Omega}$ vermittels

$$F_{\mu\Omega} = R_{\dot{\Omega}\mu\nu}{}^{\cdot\cdot\nu} \quad , \quad R_{\mu\Omega} = - R_{\dot{\Omega}\nu\mu}{}^{\cdot\cdot\nu} \quad . \quad . \quad . \quad . \quad . \quad (3.5)$$

ein, so erhalten wir

$$'F_{\mu\Omega} = R_{\dot{\Omega}\mu\nu}{}^{\cdot\cdot\nu} - A_\nu^\nu \, \partial_\mu \, \partial_\Omega \log \sigma = F_{\mu\Omega} - n \, \partial_\mu \, \partial_\Omega \log \sigma \quad . \quad . \quad (3.6)$$

$$'R_{\mu\Omega} = - R_{\dot{\Omega}\nu\mu}{}^{\cdot\cdot\nu} + A_\mu^\nu \, \partial_\nu \, \partial_\Omega \log \sigma = R_{\mu\Omega} + \partial_\mu \, \partial_\Omega \log \sigma \quad . \quad (3.7)$$

Dabei bedeutet n die Anzahl der (komplexen) Veränderlichen. (In unserem Falle ist $n = 2$ zu setzen.)

Die Grössen

$$F_{\mu\Omega} + n \, R_{\mu\Omega} \qquad \text{bzw.} \qquad R_{\dot{\Omega}\mu\lambda}{}^{\cdot\cdot\nu} - \frac{1}{n} A_\lambda^\nu \, F_{\mu\Omega} \quad . \quad . \quad . \quad . \quad (3.8)$$

sind also bei konformen Abänderung des Fundamentaltensors invariant. Die letzte Grösse lässt sich nach [56] auch in der Form

$$R_{\dot{\Omega}\mu\lambda}{}^{\cdot\cdot\nu} + \frac{1}{n} A_\lambda^\nu \, \partial_\mu \, \partial_\Omega \log \mathfrak{g}, \qquad \mathfrak{g} = |g_{\omega N}|, \quad . \quad . \quad . \quad . \quad . \quad (3.9)$$

schreiben. Aus diesen Beziehungen folgt nun der

S a t z. *Die notwendige und hinreichende Bedingung dafür, dass die Uebertragung konformeuklidisch ist, besteht darin dass*

$$R_{\dot{\Omega}\mu\lambda}{}^{\cdot\cdot\nu} = - \frac{1}{n} A_\lambda^\nu \, \partial_\mu \, \partial_\Omega \log \mathfrak{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.10)$$

ist.

B e w e i s. Die Notwendigkeit von (3.10) ist nach (3.8) klar. Andererseits geht durch die Abänderung

$$'g_{\mu\Omega} = \frac{\left| \dfrac{\partial\,(f, h)}{\partial\,(z^1, z^2)} \right|^2}{\mathfrak{g}^{\frac{1}{n}}} \, g_{\mu\Omega} \quad , \quad . \quad . \quad . \quad . \quad . \quad . \quad (3.11)$$

wo f und h zwei willkürliche analytische Funktionen der z^1, z^2 sind, deren Funktionaldeterminante nicht verschwindet, die Übertragung in eine neue über, wobei nach (1.6), (3.3) und (3.10) jetzt

$$'R_{\dot{\Omega}\mu\lambda}{}^{\cdot\cdot\nu} = R_{\dot{\Omega}\mu\lambda}{}^{\cdot\cdot\nu} + \frac{1}{n} A_\lambda^\nu \, \partial_\mu \, \partial_\Omega \log \mathfrak{g} = 0 \quad . \quad . \quad . \quad (3.12)$$

gilt.

Chemistry. — *Untersuchungen über den Fettstoffwechsel.* II. Von P. E. VERKADE und J. VAN DER LEE. (Communicated by Prof. J. BÖESEKEN).

(Communicated at the meeting of February 25, 1933.)

§ 1. In der ersten Mitteilung dieser Reihe[1]) wurde nachgewiesen, *dass bei zwei gesunden Versuchspersonen* (V. und v. D. L.) *nach Verabfolgung von Triundecylin,* von uns *Undekafett* genannt, *bedeutende Mengen Undekandisäure im Harn ausgeschieden wurden.* Im menschlichen Organismus wird also — und zwar, wie wir später sehen werden, mit grosser Leichtigkeit — die endständige Methylgruppe der Undekansäure zu einer

$$\begin{array}{c} CH_3 \\ | \\ (CH_2)_9 \\ | \\ COOH \end{array} \rightarrow \begin{array}{c} COOH \\ | \\ (CH_2)_9 \\ | \\ COOH \end{array}$$

Carboxylgruppe oxydiert. Für diesen Oxydationstypus wurde die Benennung *ω-Oxydation* eingeführt.

Selbstverständlich stellten wir uns nun zunächst die Frage, ob Verabfolgung der chemisch reinen *einfachen* Triglyceride von anderen, höheren und niedrigeren, geraden und ungeraden, gesättigten Fettsäuren ebenfalls die Ausscheidung der korrespondierenden Disäuren zur Folge hat, und im bejahenden Falle, in welchem Masze. Untersuchungen zur Beantwortung dieser Frage sind nunmehr von uns durchgeführt worden und haben unseres Erachtens zu sehr merkwürdigen und wichtigen Ergebnissen geführt, welche im Nachstehenden besprochen werden.

§ 2. Während unserer ursprünglichen Versuche mit Undekafett bildete dieses einen Teil einer Kost, die sehr arm an Kohlenhydraten war; es bekam z.B. die Versuchsperson V. damals täglich 23 g Kohlenhydrat, 136 g Eiweiss, 192 g gewöhnliches Nahrungsfett und 100 g Undekafett. Beim fortgesetzten Studium der durch Undekafett verursachten *Diacidurie* haben wir nun gefunden, dass Hinzufügung einer reichlichen Menge Kohlenhydrat zur Kost bei den meisten Personen, darunter auch den Versuchspersonen V. und v. D. L., zu einer ausgesprochenen Erhöhung der Disäureausscheidung führt. Von diesem merkwürdigen und gewiss bedeutsamen Ergebnis, das sehr vermutlich auf eine Rolle der Leber bei den sich hier abspielenden Prozessen hindeutet und in einer späteren Abhandlung ausführlich behandelt werden wird, haben wir bei den Untersuchungen, die uns jetzt beschäftigen werden, wiederholt profitiert.

---

[1]) Diese Proceedings 35, 251 (1932).

§ 3. *Trinonylin.*

Die Herstellung des benutzten Trinonylins ist schon an anderer Stelle beschrieben worden[2]). Der Erstarrungspunkt dieses Fettes war 8,7°, die Verseifungszahl 328,7 (berechnet 328,5).

Der gesunden Versuchsperson v. d. L. wurde eine Kost verabreicht, welche neben *100* g Trinonylin u.a. eine reichliche Menge Kohlenhydrat enthielt; von einer eingehenden Besprechung dieser Kost und ebenfalls derjenigen bei den übrigen hier behandelten Versuchen sehen wir in dieser Mitteilung ab. Das flüssige Trinonylin wurde als solches in Portionen von 20 g im Laufe von 10½ Stunden getrunken; auch das Kohlenhydrat wurde in Portionen und zwar jedesmal kurze Zeit vor dem Fett genommen. Von dem Zeitpunkt der Verabreichung der ersten Fettportion an wurde der Harn der Versuchsperson sorgfältig gesammelt, und zwar einschliesslich desjenigen, der am nächsten Morgen gleich nach dem Aufstehen entleert wurde. Auf Grund später zu publizierender Untersuchungen wussten wir, dass länger fortgesetztes Auffangen des Harns völlig überflüssig war.

Der gesammelte Harn wurde mittels Kalilauge stark alkalisch gemacht und darauf auf einem Wasserbade soweit wie möglich eingedampft. Einerseits findet hierbei Hydrolyse vorhandener Hippursäure statt, wodurch die weitere Aufarbeitung nur erleichtert werden kann; anderseits ist es wohl fast sicher, dass auch etwa vorhandene Verbindungen von Dicarbonsäuren mit Glykokoll, Glucuronsäure und dergl. — für deren Auftreten wir jedoch bisher niemals irgendeine Andeutung erhalten haben — hierbei hydrolysiert werden. Der Eindampfrückstand wurde nun in Wasser aufgenommen, mit Salzsäure angesäuert und lange Zeit kontinuierlich mit Äther extrahiert. Der ätherische Extrakt wurde filtriert und der nach Abdestillieren des Äthers erhaltene Rückstand in vacuo bei etwa 80° getrocknet. Dieser Rückstand wurde darauf fünfmal mit jedesmal 50 cm³ trockenem Benzen ausgekocht. Die ungeraden Dicarbonsäuren der Bernsteinsäurereihe sind, mit Ausnahme von Malonsäure, sehr gut in siedendem Benzen löslich und werden also auf diese Weise flott extrahiert; in kaltem Benzen dagegen sind sie sehr wenig löslich[3]), Azelainsäure z.B. nur 9 mg in 100 g Benzen bei 25°,0. Die aus der kalten benzenischen Lösung ausgeschiedene Krystallmasse wurde abgesogen und aus 75 cm³ siedendem Wasser unter Zusatz von etwas Entfärbungskohle umkrystallisiert. Beim Abkühlen schied sich hieraus 0,60 g reine *Azelainsäure (Nonandisäure)* aus. Der Schmelzpunkt dieses Produktes war 104—105°; der Mischschmelzpunkt mit einer in üblicher Weise aus Rizinusöl dargestellten und bei 106,5—107,5° schmelzenden Säure war 105—106°. Aus den benzenischen und wässerigen Mutterlaugen wurde noch 0,05 g derselben Säure gewonnen. Insgesamt konnte somit aus dem Harn *0,65* g Azelainsäure isoliert werden.

0,2020 g → 20,53 cm³ 0,1041 n. NaOH. Äq. gew. *94,5*; berechnet 94,1.

---

[2]) Verkade, van der Lee und Frl. Meerburg, Rec. trav. chim. **51**, 850 (1932).

[3]) Verkade und Coops, Rec. trav. chim. **49**, 578 (1930).

Ein ähnlicher Versuch wurde bei der Versuchsperson V. angestellt. Diese schied nach Genuss von *85* g Trinonylin *0,35* g Azelainsäure im Harn aus. Mit dieser Säure wurde eine Mikroelementaranalyse nach PREGL ausgeführt:

4,348 mg → 9,15 mg CO_2 und 3,21 mg H_2O : C *57,39 %* H *8,26 %*
4,491 mg → 9,48 mg CO_2 und 3,37 mg H_2O : C *57,57 %* H *8,40 %*
Berechnet für $C_9H_{16}O_4$: C 57,41 % H 8,56 %

§ 4. *Tricaprin.*

Das verwendete Tricaprin war wieder in der an anderer Stelle beschriebenen Weise hergestellt und gereinigt. Als Ausgangsmaterial diente Caprinsäure, die mittels einer auf n.Oktylbromid angewandten Malonestersynthese erhalten worden war. Tricaprin schmilzt bei 31,1°, also unter Körpertemperatur und ist daher für Versuche wie die vorliegenden ohne weiteres gut brauchbar (vergl. § 6).

Wir vermuteten — und wie sich nachstehend zeigen wird, mit Recht —, dass eine Menge von 25 g Tricaprin reichlich genügen würde um die Ausscheidung von Sebacinsäure im Harn leicht beweisen zu können; den Grund dieser Vermutung brauchen wir hier nicht zu erörtern. Daher wurden die Versuche hier in der Standardausführung vorgenommen, die wir bei anderen Arbeiten anwenden und die in einer späteren Abhandlung etwas ausführlicher besprochen werden wird:

Die gesunde Versuchsperson V. nahm kurz nach dem Aufstehen auf nüchternen Magen *25* g Tricaprin und eine reichliche Menge Kohlenhydrat zu sich. Von diesem Zeitpunkt an wurde mit Zwischenpausen von 1—2 Stunden der Harn aufgefangen und in jeder Portion auf das Vorhandensein von Sebacinsäure reagiert. Es genügt hierfür, den Harn mit etwas konzentrierter Phosphorsäure zu versetzen; bereits bei sehr geringer Konzentration an Sebacinsäure scheidet sich diese dann in charakteristischer Weise mikrokrystallinisch ab. Das Auffangen des Harns wurde solange fortgesetzt, bis zwei aufeinanderfolgende Portionen keine positive Reaktion mehr ergaben. Die Versuchsperson, die bis zu diesem Augenblick keine weitere Nahrung zu sich genommen hatte, nahm dann ihre normale Lebensweise wieder auf.

Der in der oben (§ 3) beschriebenen Weise erhaltene Ätherextrakt wurde mit siedendem Wasser extrahiert. Die heiss filtrierte Lösung wurde bis auf etwa 50 cm³ eingedampft; beim Abkühlen schied sich schon fast reine *Sebacinsäure (Dekandisäure)* aus. Diese Säure wog *0.55* g und schmolz bei 129,5—130,5°; der Mischschmelzpunkt mit einer in der üblichen Weise aus Rizinusöl hergestellten und bei 132,5—133,5° schmelzenden Säure lag bei 131—132°. Die Titration dieser rohen Säure ergab folgendes Resultat:

0,1202 g → 11,26 cm³ 0,1046 n. NaOH. Äq. gew. *102,0;* berechnet 101,1.

Bei einem anderen derartigen Versuch schied diese Versuchsperson nach Genuss von *25* g Tricaprin *0,11* g Sebacinsäure im Harn aus.

Auch bei der Versuchsperson v. d. L. wurden zwei ähnliche Versuche angestellt. Bei diesen wurde nach Genuss von 25 g Tricaprin nebst einer reichlichen Menge Kohlenhydrat, *1,10* g, bezw. *0,29* g Sebacinsäure im Harn ausgeschieden. In Anbetracht der grossen Bedeutung dieser Tatsache geben wir noch einige Analysendaten der bei dem letzten Versuch isolierten, noch einmal unter Zusatz von etwas Entfärbungskohle aus Wasser umkrystallisierten und dann bei 131,5—133° schmelzenden Säure:

0,1198 g → 11,26 cm³ 0,1046 n. NaOH. Äq.gew. *101,7*

Mikroelementaranalyse nach Pregl:

4,094 mg → 8,95 mg CO_2 und 3,22 mg H_2O : C *59,62* % H *8,80* %
4,240 mg → 9,26 mg CO_2 und 3,33 mg H_2O : C *59,56* % H *8,79* %
Berechnet für $C_{10}H_{18}O_4$: C 59,36 % H 8,98 %.

§ 5. *Triundecylin.*

Hier genügt jetzt die Bemerkung, dass unter Anwendung der obenerwähnten Standardausführung der Versuche und bei Verabreichung von 25 g dieses Fettes nebst einer reichlichen Menge Kohlenhydrat von den Versuchspersonen V., bezw. v. d. L. Mengen *Undekandisäure*, die von *0,29* g bis *0,66* g, bezw. von *0,60* g bis *1,59* g wechselten, ausgeschieden wurden. Diese Versuche werden in derjenigen Abhandlung, in welcher wir die Rolle des Kohlenhydrats behandeln, näher zur Sprache kommen.

§ 6. *Trilaurin.*

Das benötigte Trilaurin wurde mit einer Ausbeute von reichlich 85 % durch wiederholtes Umkrystallisieren von Tangkallakfett (welches von Litsea sebifera Bl. herrührt) aus einem Gemisch von 2 Vol. 96 %-igem Alkohol und 1 Vol. Äther erhalten[4]). Die Verseifungszahl betrug 263,0 (berechnet 263,5), der Erstarrungspunkt *44,75°*.

Durch Zusammenschmelzen von 1 Teil Trilaurin mit 1 Teil *Kokosfett* wurde ein Gemisch erhalten, das bei Körpertemperatur völlig flüssig wurde. Den Gebrauch eines solchen Gemisches hielten wir zur Erzielung einer sicherlich guten Resorption des Fettes für notwendig. Bei Verabfolgung von in Kaffee oder magerer Fleischbrühe völlig aufgeschmolzenem *unvermischtem* Trilaurin wurde dieses zu einem erheblichen Prozentsatz mit dem Kot in der Form grösserer und kleinerer festen Stücke ausgeschieden. Aus zwei Gründen haben wir für den genannten Zweck gerade Kokosfett und nicht irgendein bei niedriger Temperatur erstarrendes Öl verwendet. Zunächst ist es aus der Speisefettindustrie bekannt, dass gerade ein Zusatz von Kokosfett besonders geeignet ist, den Schmelzpunkt eines Fettes herabzusetzen; zweitens enthält auch das Kokosfett als Säurekomponente eine

---

[4]) Vergl. van Eldik Thieme, Dissertation, Delft, 1911. S. 26 u.f.

bedeutende Menge (etwa 50 %) Laurinsäure. Selbstverständlich kann gegen diesen Zusatz von Kokosfett der Einwand erhoben werden, dass dieses Fett als Säurekomponente noch verschiedene andere, höhere und niedrigere Fettsäuren in wechselnden, sei es denn auch immer viel geringeren Mengen enthält. Aus dem Nachstehenden wird sich zeigen, dass dieser Einwand rein theoretischer Art ist, und, unter Berücksichtigung der quantitativen Zusammensetzung von Kokosfett [5]), zugleich, weshalb dies der Fall ist.

Wir haben dieses Fettgemisch verschiedenen Versuchspersonen verabreicht, schliesslich, weil keine positiven Resultate erzielt wurden, in sehr grossen Mengen. So nahm z.B. die gesunde Versuchsperson B., die bei Verabfolgung von 25 g Triundecylin nebst einer reichlichen Menge Kohlenhydrat eine recht beträchtliche Menge Undekandisäure ausschied — die betreffenden Daten werden in anderem Zusammenhang veröffentlicht werden —, nach einem Hungertage *225* g dieses Gemisches als Bestandteil einer sehr kohlenhydratarmen Kost im Laufe von 8 Stunden und später noch einmal *190* g zusammen mit einer reichlichen Menge Kohlenhydrat im Laufe von 8¼ Stunden zu sich. Bei dem erstgenannten Experiment gelang es uns, aus dem bis zum folgenden Morgen aufgefangenen Harn einige Milligramme einer Substanz zu isolieren, die bei 118—122° schmolz und deren Mischschmelzpunkt mit einer nach WALKER und LUMSDEN [6]) hergestellten und bei 125—127° schmelzenden Dodekandisäure 119—124° betrug. Nach nochmaligem Umkrystallisieren aus siedendem Benzen schmolz diese Substanz bei 122,5—125,5°. Unter Mitberücksichtigung der Löslichkeitsverhältnisse dieser Substanz — dieselbe war kaum in kaltem Wasser und in kaltem Benzen [3]), besser in kaltem Äther und siedendem Wasser, ziemlich gut in siedendem Benzen löslich — ist der Schluss wohl berechtigt, dass es sich hier sehr wahrscheinlich um *Dodekandisäure* handelt. Für eine Bestätigung der Identität durch Titration oder Mikroelementaranalyse war die geringe Menge der erhaltenen Substanz durchaus unzulänglich. Bei dem letztgenannten Experiment wurden nur Andeutungen für das Vorhandensein eines sehr kleinen Quantums derselben Substanz erhalten.

Diese Versuche sollen an anderer Stelle ausführlicher besprochen werden.

§ 7. *Tritridecylin.*

Die Herstellung des benutzten Tritridecylins ist schon an anderer Stelle beschrieben worden [2]). Die Verseifungszahl dieses Fettes war 246,1 (berechnet 247,3) und der Erstarrungspunkt *42,65°*.

Jeder der Versuchspersonen V. und v. D. L. wurde, unter Anwendung der obenerwähnten Standardausführung der Versuche, *25* g in magerer Fleischbrühe, bezw. Kaffee vollständig aufgeschmolzenes Fett nebst einer

---

[5]) Vergl. z. B. TAYLOR und CLARKE, J. amer. chem. soc. **49**, 2829 (1927).

[6]) J. chem. soc. **79**, 1197 (1901).

reichlichen Menge Kohlenhydrat verabfolgt. In dem bis zum nächsten Morgen aufgefangenen Harn konnte keine Tridekandisäure nachgewiesen werden. Jedoch wurde auch hier ein ansehnlicher Teil des verabfolgten Fettes als grössere und kleinere feste Stücke mit dem Kot ausgeschieden.

Zur Herabsetzung des Schmelzpunktes bis unter Körpertemperatur haben wir darauf Tritridecylin mit dem gleichen Gewicht an Kokosfett zusammengeschmolzen. Aus den in § 6 besprochenen Versuchen war uns bekannt, dass nach Genuss von Kokosfett im Harn keine Substanzen ausgeschieden werden, welche bei der Isolierung in reinem Zustand von eventuell ausgeschiedener Tridekandisäure hinderlich sein könnten, sodass aus diesem Grunde gegen die Vermischung mit Kokosfett kein Einwand erhoben werden konnte. Die Versuchsperson V. nahm in der üblichen Weise *50* g dieses Gemisches zusammen mit einer reichlichen Menge Kohlenhydrat zu sich; auch hiernach war jedoch im Harn keine Tridekandisäure vorhanden.

Schliesslich wurde ein Gemisch von *25* g Tritridecylin mit 10 g Paraffinöl hergestellt; ein solches Gemisch zeigt bei 37° sehr wenig Neigung zur Krystallisation. Bei dem vorhergehenden Versuch bildete Tritridecylin nur 50 % des verabfolgten Fettgemisches; in dem soeben genannten Gemisch dagegen war Tritridecylin die einzige, durch Lipase spaltbare Verbindung. Es ist denn auch denkbar und sogar nicht unwahrscheinlich, dass bei Verabfolgung eines solchen Gemisches Tridekansäure in schnellerem Tempo resorbiert wird als bei Verabreichung des Gemisches von Tritridecylin und Kokosfett und mithin eher Aussicht auf die Anhäufung von Tridekandisäure im Blut und auf die Ausscheidung derselben im Harn bestehen wird. Die Versuchsperson V. nahm das genannte Gemisch, völlig zerschmolzen in magerer Fleischbrühe, zusammen mit einer reichlichen Menge Kohlenhydrat zu sich; das Paraffinöl übte in keinerlei Weise einen störenden Einfluss aus. In dem bis zum folgenden Morgen aufgefangenen Harn fand sich auch jetzt wieder keine Tridekandisäure.

§ 8. Wie aus dem Vorstehenden erhellt, *führt die unter möglichst übereinstimmenden Umständen erfolgende Verabreichung einer Reihe einfacher Triglyceride in sehr verschiedenem Masze zu Disäureausscheidung.*

Eine sehr beträchtliche Disäureausscheidung fanden wir allein nach Verabfolgung von Triundecylin und Tricaprin. Bei Verabreichung einer immer gleichen Menge desselben Triglycerids an dieselbe Versuchsperson unter äusserlich gleichen Verhältnissen kann die Menge ausgeschiedener Disäure ziemlich stark wechseln, während sich auch verschiedene Versuchspersonen gegenüber demselben Triglycerid sehr verschieden verhalten können. Auf diese Tatsachen, bezüglich deren oben schon einige Daten zu finden sind, gehen wir in späteren Abhandlungen ein. Es ist infolgedessen schwierig, jetzt schon anzugeben, ob Triundecylin oder Tricaprin am meisten zu Disäureacidosis und Disäureausscheidung Anlass gibt, m.a.W. — wie wir

dies bezeichnen werden — am stärksten *diacidogen* ist; wir vermuten, dass dies seitens des erstgenannten Fettes der Fall ist. Tricaprin ist dann etwas weniger und Trinonylin ist fraglos bedeutend weniger diacidogen. Bei den beiden von uns untersuchten höheren Triglyceriden, Tritridecylin und Trilaurin, wurde, trotz ihrer Verabfolgung an stark zu Diacidurie neigende Versuchspersonen Disäure gar nicht, bezw. höchstens in Spuren im Harn angetroffen.

Eine selbstverständlich vorläufige, vergleichende Übersicht der diacidogenen Eigenschaften der untersuchten Triglyceride gestaltet sich folgendermassen:

	Termnummer [7])		
Tritridecylin	12	nicht	diacidogen
Trilaurin	11	nahezu nicht	,,
Triundecylin	10	stark	,,
Tricaprin	9	etwas weniger	,,
Trinonylin	8	ziemlich schwach	,,

Wie rätselhaft dieser merkwürdige Verlauf der diacidogenen Eigenschaften zum guten Teile noch sein möge, Eines ergibt sich wohl sehr deutlich, nämlich *dass von einem Gegensatz in dieser Hinsicht zwischen den von geraden und ungeraden, d.h. von „körpereigenen" und „körperfremden" Fettsäuren abgeleiteten Triglyceriden zweifellos keine Rede ist*; man vergleiche das Verhalten der Triglyceride mit den Termnummern 9 und 10, bezw. 11 und 12. Ohne viel Risiko lässt sich nun eine Prognose betreffs des Verhaltens anderer, höherer und niedrigerer, einfacher Triglyceride unter gleichen Umständen stellen. Bei den höheren Triglyceriden — wir lassen hier die Schwierigkeiten, welche diese Substanzen infolge ihrer hohen Schmelzpunkte bieten würden, ausser Betracht — würde von Disäureacidosis und Disäureausscheidung fraglos keine Rede sein. Dagegen erscheint es uns, was die niedrigeren Fette betrifft, keineswegs unwahrscheinlich, dass es möglich sein wird, bei Verabfolgung einer grösseren Menge *Tricaprylin* (Termnummer 7) an eine stark zu Diacidurie neigende Versuchsperson, ohne Mühe aus dem Harn *Korksäure (Oktandisäure)* zu isolieren. Wir werden die diesbezügliche Untersuchung dieses Triglycerids in der nächsten Zeit in Angriff nehmen.

§ 9. In folgenden Abhandlungen werden wir nachweisen, dass die geraden und ungeraden Fettsäuren ausser längs dem klassischen, von KNOOP gefundenen Wege der β-Oxydation, die wir fortan als *einseitige β-Oxydation* bezeichnen werden, zugleich noch auf eine andere Weise abgebaut werden: *indem nämlich durch ω-Oxydation der Fettsäure die entsprechende Disäure gebildet wird, welche dann über niedrigere, durch zweiseitige β-Oxydation gebildete Disäuren als Zwischenprodukte weiter*

[7]) VERKADE und COOPS, Rec. trav. chim. **49**, 569 (1930).

abgebaut wird. Beiläufig sei hier bemerkt, dass wir die Möglichkeit des Vorkommens noch anderer Abbaumechanismen keineswegs ausschliessen möchten.

Die oft sehr reichliche Ausscheidung von Disäure nach Genuss von Triundecylin und Tricaprin, besonders die im letztgenannten Falle, weil doch dieses Fett von einer natürlichen Fettsäure mit gerader Zahl von Kohlenstoffatomen abgeleitet ist, ist unseres Erachtens übrigens an sich schon ein starker Fingerzeig, dass diese Disäuren eine Rolle bei einer bis jetzt unbekannten Abbauweise der Fettsäuren spielen; die Annahme, dass bei dem Katabolismus dieser Substanzen so grosse Mengen Disäure als nutzloses Nebenprodukt entstehen sollten, ist doch zweifellos gar nicht akzeptabel.

Selbstverständlich werden wir uns dann später auch mit der Frage beschäftigen müssen, *infolge welcher Umstände* Verabreichung der verschiedenen einfachen Triglyceride unter möglichst übereinstimmenden Verhältnissen an dieselben gesunden Versuchspersonen in so typisch verschiedenem Grade zur Anhäufung von Disäure im Blute und Ausscheidung derselben im Harn führt. Gegenwärtig ist diesbezüglich noch sehr wenig mit Bestimmtheit zu sagen. Die Verarbeitung von per os verabreichtem Fett geschieht durch einen höchst verwickelten Komplex von Prozessen, den wir sogar in grossen Zügen noch keineswegs gut überblicken können, und dessen einzelne Phasen durchweg wohl mit einer für jedes Triglycerid, bezw. jede Fettsäure charakteristischen Geschwindigkeit durchlaufen werden. Die Annahme liegt nun sehr nahe, dass es von den Geschwindigkeitsverhältnissen aller dieser Prozesse abhängig sein wird, ob, und bejahendenfalls in welchem Masze nach Verspeisen eines bestimmten Triglycerids Anhäufung von Disäure im Blut auftritt und infolgedessen — vielleicht bei Überschreitung einer bestimmten Konzentration — Ausscheidung hiervon durch die Niere. Versuche wie die in dieser Abhandlung beschriebenen können nur zu einer ersten Orientierung über die neu entdeckte Abbauweise der Fettsäuren und die damit zusammenhängenden Erscheinungen dienen. Ein tieferer Einblick ist nur durch das Studium bestimmter einzelner Phasen des Gesamtprozesses des Fettstoffwechsels zu erlangen. Wir denken hier u.a. an eine vergleichende Untersuchung über die Verseifungsgeschwindigkeit der geraden und ungeraden einfachen Triglyceride, im besonderen unter Verhältnissen, die möglichst viel den im Darm herrschenden Verhältnissen ähneln, also unter Einfluss von Pankreaslipase, Gallensäuren usw.[8]; an Versuche mit der Verabfolgung per os von Glyceriden der Dicarbonsäuren, eventuell von löslichen Salzen dieser Säuren und der Fettsäuren; an Injektion von Lösungen von Salzen der Fettsäuren und der Dicarbonsäuren; an Durchströmungen überlebender Organe mit solchen Lösungen. Untersuchungen in diesen und ähnlichen Richtungen behalten wir uns vor; das erstgenannte Problem ist in unserem Laboratorium bereits von Dr. DE WILLIGEN in Angriff genommen worden.

8) Vergl. TERROINE, Ann. des sciences naturelles, Zoologie, (10) 4 (1920).

Beiläufig sei hier bemerkt, dass bei Annahme der vorstehend sehr kurz angedeuteten Auffassung bezüglich der Ursache des eigentümlichen Verlaufes der diacidogenen Eigenschaften bei den aufeinanderfolgenden Triglyceriden logischerweise auf die ebenfalls vorhandene Existenz irgend eines Verlaufes in den *ketogenen Eigenschaften* der Fettsäuren mit gerader Zahl von Kohlenstoffatomen, bezw. der hiervon abgeleiteten Triglyceriden geschlossen werden darf oder besser werden muss. Die Frage, ob Verabfolgung verschiedener natürlicher Fette oder Öle unter möglichst gleichen Umständen an dieselbe Versuchsperson in verschiedenem Masze Ketosis und Ketonurie zur Folge hat, wurde in der Literatur schon wiederholt beleuchtet und wird von einigen Untersuchern bejahend, von anderen verneinend beantwortet. Wir werden die diesbezügliche Literatur hier nicht besprechen. Von Untersuchern der erstgenannten Kategorie wird mehrmals als Resultat ihrer Versuche mitgeteilt, dass nach Genuss von Butter eine grössere Menge Ketonkörper im Harn ausgeschieden wird als nach Genuss der gleichen Menge eines Fettes mit einem kleinen Gehalt an niederen Fettsäuren; dies scheint uns in der Tat nicht unmöglich. Auch hier besteht wieder eine Möglichkeit für interessante und zweifellos wichtige Untersuchungen mit vollkommen definierten, synthetischen Fettsäurederivaten. Wir sind uns jedoch davon bewusst, dass es entschieden sehr schwer sein wird, auf diesem Gebiete wirklich unzweideutige Resultate zu erhalten.

§ 10. Wie bereits in der vorigen Abhandlung [1]) dargelegt wurde, war der ursprüngliche Zweck unserer Untersuchungen Beiträge zur Kenntnis des Katabolismus der von Fettsäuren mit ungerader Zahl von Kohlenstoffatomen abgeleiteten Fette zu liefern. Behufs dieser Untersuchungen stellten wir absichtlich ein Fett her, das als Säurekomponente *nur eine einzige* solche Fettsäure enthielt; hierdurch könnte die Isolierung in reinem Zustande und die Identifizierung eventuell im Harn ausgeschiedener partieller Abbauprodukte der Säurekomponente möglicherweise erleichtert werden. Wir wählten als solche die *Undekansäure*, eigentlich allein, weil diese Säure leicht in grossen Mengen rein zu erhalten ist und das entsprechende Triglycerid unter Körpertemperatur schmilzt. Diese Wahl erweist sich nun als eine sehr glückliche; ist doch unter den betreffenden Fetten *Triundecylin* dasjenige mit den weitaus stärksten diacidogenen Eigenschaften. Bei Gebrauch eines von einer höheren Fettsäure mit ungerader Zahl von Kohlenstoffatomen abgeleiteten Fettes wären wir der ω-Oxydation wohl so gut wie sicher nicht auf die Spur gekommen und hätten sich unsere Untersuchungen vielleicht nicht auf solche über den Fettstoffwechsel im allgemeinen ausgedehnt. In diesem Zusammenhang sei auf eine Besprechung der von Anderen mit Intarvin und Diafett angestellten Untersuchungen in der nächstfolgenden Abhandlung hingewiesen.

In anderer Hinsicht ist die betreffende Wahl jedoch die ungünstigste gewesen, die möglich war, wenn wir nämlich auf die Möglichkeit therapeu-

tischer Anwendung solcher Fette, z.B. bei der Behandlung des Diabetes, achten. In der vorigen Abhandlung wurde nachgewiesen, dass unter den bei den dort besprochenen Untersuchungen gegebenen Umständen der teilweise Ersatz von Nahrungsfett durch Undekafett eine sehr deutliche Verminderung der Ketosis zur Folge hat. Es besteht a priori keinerlei Grund zu der Annahme, dass dies bei Verabreichung von Triglyceriden anderer Fettsäuren mit ungerader Zahl von Kohlenstoffatomen nicht der Fall sein wird. Aus unseren hier beschriebenen Untersuchungen zeigt sich nun, dass die niedrigeren Triglyceriden in viel geringerem Grade als Undekafett und die höheren überhaupt nicht zu Disäureacidosis führen. *Aus diesem Grunde sind diese Produkte für therapeutische Anwendungen entschieden dem Undekafett vorzuziehen.* Auch in diesem Zusammenhange verweisen wir nach der Besprechung von Literatur über Intarvin und Diafett in der nächsten Abhandlung. In einem für medizinische Zwecke ins Auge gefassten derartigen Fett wird Undekansäure als Säurekomponente am liebsten fehlen müssen. Wir hoffen bald auch zu klinischen Untersuchungen mit hierfür am besten geeigneten Fetten Gelegenheit zu haben.

Gern entledigen wir uns schliesslich noch der Dankespflicht gegenüber der Verwaltung des „HOOGEWERFF-Fonds" und der „ERASMUS-Stichting" zu Rotterdam für die von diesen Seiten gewährte Unterstützung für unsere Untersuchungen.

Rotterdam, Laboratorium der Nederlandsche Handels-Hoogeschool.

Februar 1933.

Geology. — *The Malay double (triple) orogen.* By G. L. SMIT SIBINGA (Communicated by Prof. G. A. F. MOLENGRAAFF).

(Communicated at the meeting of March 25, 1933).

PART II.

The Australian double-orogen.

According to several geologists there exist geological relations and connections between the Halmaheira Archipelago, especially its Eastern part and New Guinea. Others have joined parts of the Pelew-orogen and of the Australian orogen to tectonical units. In the writer's opinion the available data, as far as they may be considered to be well established geological facts, do not support these views.

There exists a striking structural analogy in the main features between the Australian double-orogen, marginal to the Northern border of the Australian continent and the Asiatic double-orogen marginal to the Southern border of the Asiatic continent. The analogy is a principal one, though gradually there are great differences.

North of the Australian continent two orogens (the South- and the North-New Guinea-orogen) already well developed in Pretertiary time may be discerned. The S.-New Guinea-orogen may be pursued from the Louisiade Archipelago through whole New Guinea up to the group of islands: Salawati, Batanta, Waigeu. The Kumamba Is. and the Cyclope-Bougainville range belong to the N.-New Guinea-orogen, the prolongation of which is still uncertain. The available geological data neither allow to fix the orogenetic nor the magmatic cycles of these Australian orogens.

The geosyncline situated between the S.-New Guinea-orogen and the Australian continental border, which still existed in Miocene time has been very intensively folded during Post-miocene time. The most prominent element of this late Neogene orogenesis are the Snow Mountains. On account of analogy with the Himalayas and with regard to the exclusively Northern dip of the strata nearly up to their central part it has been suggested by several geologists (BROUWER, FEUILLETEAU DE BRUYN, ZWIERZYCKI, a.o.) that overfolding and overthrust sheets directed to the South occur in the Snow Mountains. If this view would appear to be right a strong Southward surface reaction would bear witness to a corresponding strong Northward continental action. This is an important gradual difference with the folding of the Neogene geosyncline between the Sunda-orogen and the Southern Asiatic continental border, which is much less intensively folded and which does not show strong surface reactions of any vigorous continental action.

The geosyncline between the S.- and N.-New Guinea-orogen has been strongly folded in Plio-pleistocene time only in its Northern part though less intensively than the Southern one. Overturned folds and small over-thrusts directed to the North however show a still distinct surface reaction of a Southward directed counteraction of the N.-New Guinea-orogen and the Pacific oceanfloor. Here we may also notice a great gradual difference with the geosyncline between the Sunda- and Molucca-orogen South of the Asiatic continent, which still finds itself in a geosynclinal stage.

Apart from great principal analoga the Australian double-orogen especially the interjacent geosynclines show important gradual differences in their orogenetic evolution compared with the Asiatic double-orogen South of the Sundaland. That the orogenetic development of the Australian double-orogen has proceeded much further may be reasonably explained by the much stronger Australian continental action with Northward tendency during Tertiary time in contradistinction with the slight or perhaps entirely absent Asiatic Tertiary action with opposite tendency.

Crustmovements.

WEGENER [13]) considered the island arcs off the Eastcoast of Asia as marginal ranges separated from the continent and stuck in the simatic oceanfloor by the westdrift of Asia. This explanation of the origin of the Asiatic arcs is not adequate in the light of present knowledge, for it depends on the westward drift of Asia, which at present seems highly improbable.

TAYLOR [14]) e.g. showed conclusively that the distribution of the crustmoving forces is intimately related to latitude and to hemispheres. The continents in the Northern hemisphere moved in Southerly direction (disjunctive basins around the North Pole), while those in the Southern hemisphere moved in Northerly direction (antarctic disjunctions). The crustal movements were radial and dispersive from both polar regions and consequently the Asiatic continent slid Southward with a strong deflection to the East.

The inadequacy of the Asiatic westdrift has been stressed particularly by MOLENGRAAFF [15]) in connection with the nature and origin of the mid-atlantic ridge. The recent gravity measurements of VENING MEINESZ in the East Indian Archipelago (l.c.) also point to a Southeast instead of a West tendency of the Asiatic continent.

This conception seems to be contradictory to the Youngtertiary tectonic plan bordering on the Malay earthlobe (Sundaland), which shows no evidence in support of a strong Youngtertiary Asiatic earthmovement at all.

The tectonic processes at the crust's surface may be studied exactly in plan, in profile only approximately, whereas only a very thin surface layer is accessible to direct observation. Surface and subsurface structure may be quite incongruent, particularly in complicated area's as the East Indian Archipelago. This incongruity has still been emphasized by the recent maritime gravity survey.

We know that superficial folds originate through compressional stress. At the best we may ascertain its direction exactly in the surface layers. But we know also that the observed compressional stress at the surface has its seat in deeper layers of the earth's crust and that here the direction of stress may be quite divergent. For a definitely directed stress in the surface layers may be induced in the depth in a number of different ways: it may be an equally directed stress, an obliquely directed (differential shear) or even an inversely directed one (underthrusting). To the latter especially attention should be called.

---

13) A. WEGENER. Die Entstehung der Kontinente und Ozeane. 4te Aufl. 1929. p. 201.

14) F. B. TAYLOR. Sliding continents and tidal and rotational forces. in: W. A. J. M. v. WATERSCHOOT v. D. GRACHT, etc. Theory of continental drift. 1928. p. 158.

15) G. A. F. MOLENGRAAFF. Wegener's continental drift. in: idem p. 90.

G. A. F. MOLENGRAAFF. The coral reef problem and isostasy. These Proceedings Vol. 19. 1916. p. 610.

A point of first-order importance in this connection is the actual movement of the crust being involved. It is usually assumed that the overthrust part is actually moving. The opposite or rather supplementary idea of underthrusting has been emphasized by MOLENGRAAFF [16]) and HOBBS [17]) and lateron by others.

If movement takes place along a thrustplane and the overriding (overthrust) part of the crust is assumed to be moved in space, there is as much reason to assume that the overridden (underthrust) part of the crust may be moved as well, but in the opposite direction. From a mechanical point of view however there is much more reason to assume that the underthrust mass, which is by far the main part of the orogenic body, actually moved than the relatively thin surface layer. Studying surface structures in the right proportion to their subsurface negatives even prominent features as extensive overfolds and overthrusts appear to be but surface reactions of the really deepseated action. Therefore the subsurface protuberance of an orogen should be more accurately called the negative orogen, realizing that this negative orogen from a geotectonic viewpoint is the most important part of the orogen, being actually the cradle of orogenetic action and evolution, of which the geologist only observes the surface reaction in the positive orogen.

Now predominant positive tectonic features as geanticlines and geosynclines, extensive overfolds and overthrusts require considerable relative movements and consequently a conspicuous negative action. Moreover they always will have their main trend more or less normal to the direction of the deepseated stress that gave rise to them. Consequently they are the most suitable to compute the intensity and the direction of the subsurface action of the negative orogen.

Reckoning with all these factors it will be possible to draw to a certain extent conclusions from the surface reaction with regard to the subsurface orogenetic action i.c. the continental crustmovement.

The tectonic plan of the Neogene geosyncline bordering on the Sundaland has recently be discussed by the writer, which here may be referred to [18]). The main phase of Young-tertiary orogenesis acted much less intensively at the Asiatic border than at the Australian. Conspicuous surface reactions as extensive overfolding and overthrusting directed towards the Sundaland, which could bear witness to an important Young-

---

16) G. A. F. MOLENGRAAFF. Folded mountain chains overthrust sheets and blockfaulted mountains in the East Indian Archipelago. Compte Rendu du XII Congrès Géol. Intern. Toronto 1913, p. 689.

G. A. F. MOLENGRAAFF. l.c. [12]), p. 300.

17) W. H. HOBBS. Mechanics of formation of arcuate mountains. Journal of Geology XXII. 1914, p. 71.

18) G. L. SMIT SIBINGA. The Tertiary virgations on Java and Sumatra, their relation and origin. These Proceedings Vol. 35. 1932, p. 584.

G. L. SMIT SIBINGA. The interference of meridional and transversal stress in the Southeastern part of Borneo. Idem, p. 1090.

tertiary Asiatic action up till now have not been observed. However the Tertiary structures on Sumatra-Java are in the main directed towards the Sundaland. Though there is apparently no evidence for any strong Tertiary Asiatic action the minor tectonic features on Sumatra-Java do not speak against a Tertiary Asiatic action, be it a subordinate one (in the writer's opinion better called continental reaction).

On the other hand it may be emphasized that in Tertiary time the sphere of very strong Australian action reached up to the border of the Sundaland, overshadowing the influence of a perhaps feeble Asiatic action, so that it seems hardly possible to disconnect both actions.

During an earlier (Cretaceous) mountain building process however, as the Archipelago did not yet find itsef in the Australian sphere of action and an Asiatic crustmovement still found an unobstructed path, there seems to be more evidence for a strong Asiatic action as advocated by TAYLOR, MOLENGRAAFF and VENING MEINESZ. Cretaceous orogenesis acted very intensively on Sumatra-Java. Overthrust sheets are considered to be highly probable as well on Sumatra (TOBLER) as on Java (HARLOFF). If the existence of great overthrust masses on these islands, directed towards the Sundaland should be proved, there is indeed strong evidence for an important Young-cretaceous Asiatic crustmovement with Southern tendency.

A vigorous Young-cretaceous Asiatic action, perhaps followed by a feeble Tertiary one, the latter being overshadowed by an intensive Young-tertiary Australian action would give a reasonable explanation for the curious fact that Young-cretaceous orogenesis is much stronger along the Asiatic — and that Young-tertiary orogenesis is much more intensive along the Australian border. The writer therefore is still inclined to award a leading part to the Australian crustmovement in the Tertiary tectonic disturbance of the Archipelago. The following arguments may further support this view.

In a former chapter it has already been discussed that the much more advanced stage of Young-tertiary orogenetic evolution of the Australian double-orogen, especially of the interjacent geosynclines in comparison with the Asiatic double-orogen South of the Sundaland points to a strong Australian action with Northern tendency.

The surface reactions in the Molucca-orogen opposite the Sahulbank show likewise strong evidence for Australian action. On Timor extensive overfolding and overthrusting of alpine character directed towards the Southeast has been discovered by MOLENGRAAFF and WANNER. WEBER's observations on Jamdena and the Kei Is. seem to point to eastward directed overthrusts, therefore also towards the Sahulbank. On Ceram northward directed isoclinal overfolding and overthrusting has been determined by RUTTEN and HOTZ, likewise towards the Australian continent. From Timor up to Ceram thus strong centrifugal surface reactions prove decidedly strong centripetal subsurface action. Moreover the Australian action is

shown conclusively by the Young-tertiary tectonic plan. The Molucca-orogen along the Australian border not only corresponds exactly with the continental contourline, but is entirely crushed and crumbled to pieces by the action of this continent. The writer already pointed out in a previous paper (l.c. 3) that the way in which the Molucca-axis has been disturbed and squeezed could only proceed from centripetal Australian action.

All these phenomena as extensive overfolding and overthrusting, squeeze and crush are totally lacking at the Asiatic continental border. Comparing the Young-tertiary tectonic plan of the Asiatic and the Australian marginal belts a sharper contrast is hardly conceivable.

Of argumentative force finally is the well established fact that the main phase of Tertiary orogenesis started with great intensity at the Australian border in the early Miocene, to be transmitted with diminishing force during the youngest Tertiary towards the Asiatic border, whose marginal belts have been folded much less vigorously in the Plio-pleistocene. This striding away of the Young-tertiary main orogenesis from the Australian to the Asiatic continent in time and space, recently clearly illustrated cartographically by UMBGROVE [19]), proves conclusively that the Australian crustmovement played a leading part in the Young-tertiary orogenesis of the Archipelago.

The writer fully agrees with VENING MEINESZ that the principal strip of negative anomalies, i.e. for the greater part the still highly uncompensated Molucca-orogen, which also in his opinion reasonably may be expected to continue along all Eastasiatic orogens up to the Aleutians is of Asiatic origin, caused by a Southeast tendency of the Asiatic continent. However the Asiatic crustmovement already set in during Pretertiary times. We know that Cretaceous orogenesis has been very strong in Eastern Asia. The gigantic Eastasiatic geosyncline has been subjected to compression since at all events late Mesozoic time. It appears that an intimate relation existed between the Pretertiary Asiatic crustmovement, the intensive Cretaceous Eastasiatic orogenesis and the compression of the active geosyncline bordering on the Asiatic eastcoast, almost all in the same period. It goes without saying that all these intensive and extensive processes gave rise to strong and widespread gravity anomalies. The gravity measurements have taught that orogenetic activity may take place without maintenance of isostatic equilibrium which implies the discernment of the temporal permanence of isostatic anomalies (c.f. last chapter). In the writer's opinion the strip of strong negative anomalies in question is one of them and must likewise be of Pretertiary origin, though in the Malay Archipelago during the Tertiary period modified and intensified by strong Australian action.

This conception harmonizes with all observed and well established geological facts.

[19]) J. H. F. UMBGROVE. Het neogeen in den Indischen Archipel. Tijdschr. Kon. Ned. Aardr. Gen. Dl. 49. 1932, p. 769.

Tertiary structural plan of the Molucca-orogen.

As stated above the main phase of Youngtertiary orogenesis started in the Molucca-orogen at the Australian continental border (Sahulbank) after the sedimentation of the Oligocene. As a rule the Youngtertiary main trends both in the Sunda-orogen and in the Molucca-orogen generated originally more or less parallel to the trends of the orogenetic axes and deviated lateron through the fracturing and bending of the latter. The writer recently pointed out that the complementary branching of trend lines (virgations) on Sumatra-Java may be explained by the bending and subsequent fracturing and faulting of the orogenetic axis (l.c. 18). But it may also have happened that the trend of the orogenetic axis could not adapt itself to the direction of compressional stress (Sumba), in which case the trends originated more or less divergent to the axis.

Considering the present structural plan it is clearly visible that the positive orogenetic axis opposite the Sahulbank is often cut obliquely by the negative axis, indicated by the general course of the island arc. The divergency may be considerable as e.g. on Babber, where the Youngtertiary trend and the positive orogenetic axis run nearly perpendicular to the negative axis. Such apparently abnormal trends are essentially not abnormal if we realize that the disturbing crush of the continental crust-movement has subsequently broken and has bent the more rigid positive orogen in several places to a much greater extent than the more plastic negative orogen. One always has to conceive the position of both axes during the main phase of Tertiary folding in question with regard to the prevailing direction of stress.

The divergent trends in the Molucca-orogen opposite the Sahulbank may reasonably be explained by subsequent fracturing and bending of the positive orogenetic axis, the axis of the more plastic negative orogen remaining still fairly undisturbed. Imagining the positive axis in its original position, still more or less indicated by the negative axis, the trend appears to be quite normal to the direction of stress.

In this complicated area the directions of stress and movement in both parts of the orogen have to be sharply discerned to get a clear notion of the whole tectonic process. As already pointed out in a previous chapter the direction of stress may be different to the direction of movement in the positive orogen, while it may be quite different too in the negative orogen. Moreover the direction of stress and movement in both parts of the orogen may be divergent to each other. Consequently at least four different factors with regard to stress and movement in an orogenic body have to be reckoned with, apart from the continental crustmovement. To simplify the matter as much as possible the minor tectonic features are to be left out of account for the time being and considerations are confined to predominant tectonic features, which always have a trend more or less normal to the direction of stress so that the directions of stress and movement pretty well

coincide, though they may be opposite to each other in the positive and the negative orogen.

As already stated above, the positive Molucca-orogen from Timor up to Ceram has been moved centrifugally towards the Australian continent, it being the surface reaction of a centripetal movement of the negative orogen due to centripetal continental stress induced by continental action with northward tendency. The centripetal continental stress is suggestively demonstrated by the tectonic plan, particularly by the shape of the crushed positive orogenetic axis.

Our present knowledge of Tertiary tectonis on Buru and the Tukang Besi Is. is still too scanty to determine the directions of movement and stress. Some more data are available about the Buton Archipelago and Eastern Celebes.

BOTHÉ's [20]) representation of the geological structure of North Buton points to eastward directed isoclinal overfolding and overthrusting. According to BROUWER (l.c. 11) Eastern Celebes may be considered as a Tertiary folded range with its convex side to the West and with a general trend, which bends approximately in the direction of the present Northeastern and Southeastern peninsulas. In this Tertiary folded range imbricated structures with "schuppen" of partly brecciated ultrabasic rocks and of limestones thrust one over the other seem to prevail, though a general direction of thrusting could not yet be determined exactly.

In the Northern part of Eastern Celebes KOOLHOVEN [21]) discovered likewise overthrusts of alpine character. KOOLHOVEN's information for two reasons is of great importance, firstly as he had at his disposal the still unpublished results of L. v. LÓCZY and his fellow-workers, who investigated the most complicated adjoining territory and secondly as he was able to determine for the first time the position of the root of the overthrust due to the favorable circumstance that in the Northern part of Eastern Celebes the overthrust mass still coheres with its root. Up till now the roots of the overthrust sheets in the Molucca-orogen have been unknown and nowhere observed. In the part bordering on the Australian continent, on account of the direction of thrusting, the region of the roots has been presumed to be situated at the inner (Asiatic) side. This presumption has now been confirmed, as in the part of the Molucca-orogen which now forms the Northeastern peninsula of Celebes the root really has been observed to be situated on the Asiatic side of the orogen. This seems to apply to Buton too, the Southern prolongation of Eastern Celebes.

On account of its marginal position at the Asiatic side of the Molucca-orogen it seems reasonable to expect the root to be continued from the

20) A. CHR. D. BOTHÉ. Voorloopige mededeeling betreffende de geologie van Z. O: Celebes. De Mijningenieur. 1927. p. 97.

21) W. C. B. KOOLHOVEN. Verslag over een verkenningstocht in den Oostarm van Celebes en den Banggai-Archipel. Jaarb. Mijnwezen 1929. Verh. p. 187.

Northern part of Eastern Celebes through the Southern part of Eastern Celebes up to Buton.

The direction of thrusting on East Celebes and Buton — where it could be determined — appeared to be the same as in the part of the Molucca-orogen bordering on the Australian continent, namely towards the outer (Australian) side of the orogen. The positive Molucca-orogen from Buton up to the peninsula of Bualemo of Northeastern Celebes thus shows a strong surface reaction indicating a corresponding vigorous action of the negative orogen directed towards the Asiatic continent.

The main phase of Tertiary orogenesis on Buton took place as in the part of the Molucca-orogen bordering on the Australian continent after the deposition of the Oligocene, in the Northern part of Eastern Celebes somewhat later, namely after the sedimentation of the Lower-miocene.

Our present knowledge of Western Halmaheira does not yet allow us to determine any direction of stress nor movement.

The part of the Molucca-orogen Southwest of Sumatra (Mentawei Is.) seems to be developed quite normally.

Geology. — *The formation of the atolls in the Toekang Besi-group by subsidence.* By PH. H. KUENEN. (Communicated by Prof. G. A. F. MOLENGRAAFF).

(Communicated at the meeting of March 25, 1933).

The present author in preparing a report on his investigations on coral reefs, as geologist of the oceanographical expedition to the East Indies on board Hr. Ms. Willebrord Snellius, came across a number of arguments in favour of the formation of atolls through upgrowth, during slow sinking of the substratum, on which they grow. It remains probable, that glacial control influenced the upper story of these great buildings, especially in aiding the levelling and regulation of the depths of the lagoon-bottoms. In this respect the classical study of MOLENGRAAFF on the influence of pleistocene lowering of sealevel in the East Indies (bibl. 4) remains untouched. In the production of the deeper mass below the living reefs and lagoons, however, the sinking of the substratum has played the principle part in the manner advocated by DARWIN, DANA and DAVIS. The group of atolls in which this can be most clearly demonstrated is the Toekang Besi-group to the south-east of Celebes. In the following paper a short summary will be given of the reasons for favouring this theory. For a detailed argumentation the reader is referred to the shortly forthcoming publication in the reports of the Snellius expedition.

ESCHER showed in 1920 (bibl. 1) that the atolls and the islands in the Toekang Besi-group are situated in rows. The first row in the south-west

is formed by atolls, the second by elevated islands, the third again by atolls and the fourth and last row again by elevated islands. ESCHER pointed out, that this arrangement must be the consequence of tectonic forces in the substratum, the islands corresponding with axes of elevation, the atolls with axes of depression. MOLENGRAAFF adopted the same view (bibl. 4).

RUTTEN (bibl. 5) pointed out, that all the rows may be anticlines, but that the atoll-anticlines were elevated less and became submerged on a general sinking of the whole region. Before starting on our expedition I decided to have a number of soundings taken, in order to ascertain whether the atolls are placed on anticlines or in synclines (bibl. 3) HETZEL (bibl. 2) visited some of the islands and came to the conclusion, that the strike of the tertiary strata is at right angles to the trend of the rows of islands and atolls. He believes, therefore, that after a folding at right angles to the rows a later block-faulting caused the formation of rows of islands and atolls, also on account of the absence of warping in the elevated reef-terraces of the islands.

Several sounding-sections were made in the group. These showed the following relations to exist (fig. 1 and plate, fig. 3).

The south-western, oblong atolls are situated on a deeply submerged ridge from which they rise abruptly. The round atolls in the centre of the group, to which the lagoon-less reefs with sand cays must also be counted [1]), are placed arbitrarily on a nearly flat plateau with dephts of about 1000 meters. Only the most southeasterly reef may be founded on a distinct ridge running north. The southwestern row of islands is continued to the southeast by some submerged banks, but there is no clear anticlinal ridge. The island Roendoema in the northeast lies on a ridge, that runs parallel to the other rows and also bears one reef. Important details of the structure were also found in the form of submarine faultscarps, that occur especially in the neighbourhood of the southernmost island, Binongko. A short excursion on the island Kaledoepa showed that the strike is here parallel to the trend of the row of islands and if we interprete the measurements made by HETZEL on WANGI WAGI in the northwest as pitch of the anticline, then the structure of the substratum corresponds to the trend in the whole group. No indications are to be found in the chart of a former trend from south-west to north-east.

Since the elevation of the islands above sealevel the doming has ceased, for as HETZEL pointed out, the elevated reef terraces are not warped. I made accurate measurements on 6 sections of the island Binongko, in the south, of the heights of the terraces. This showed that not only were they flat and straight, but also that while they were elevated 200 meters, they remained perfectly horizontal. This, together with the submarine faultscarps, the disconnected, block-like row of elevations formed by the

[1]) Some of these were wrongly taken by ESCHER as representing elevated islands and were therefore included in his northeastern row of islands.

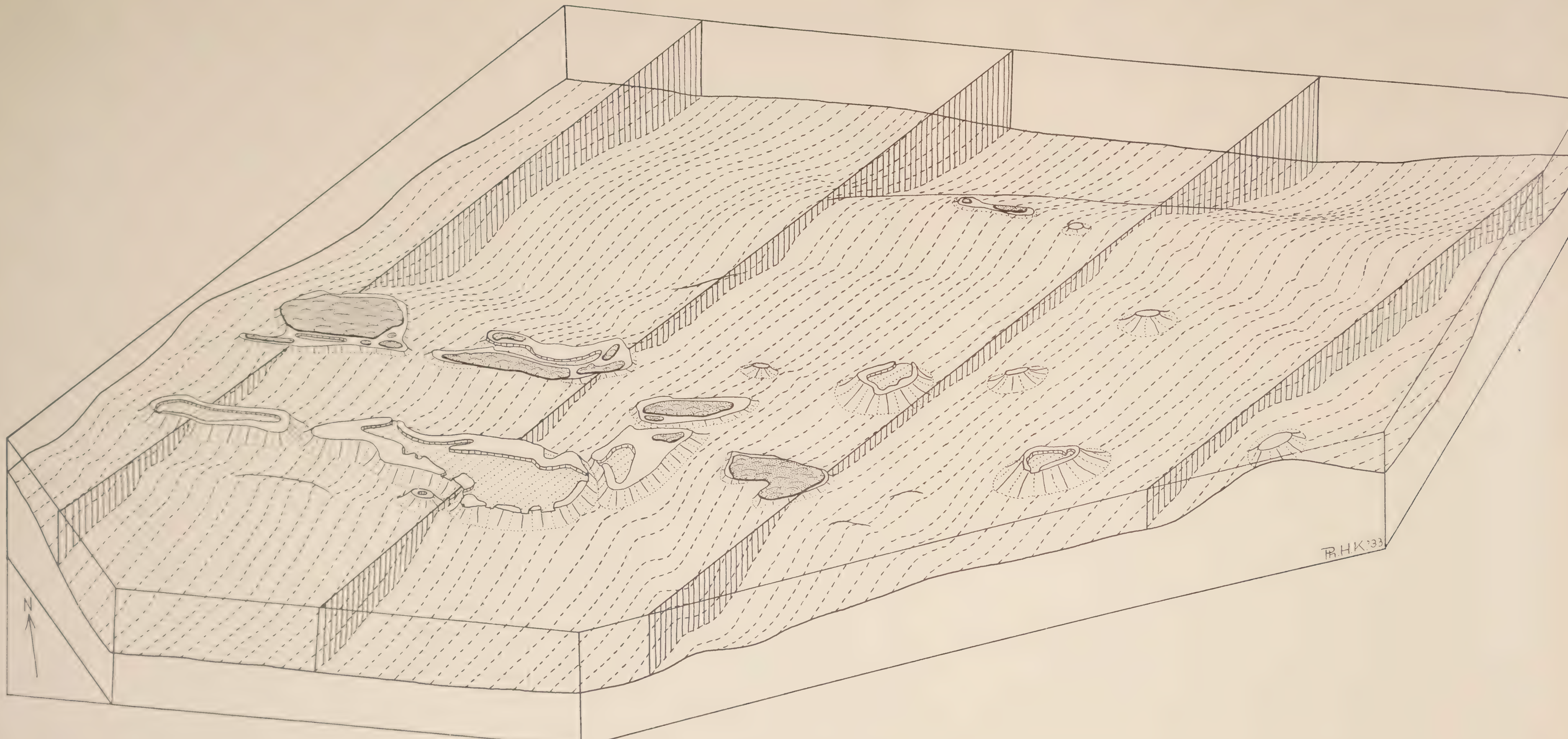


Fig. 3. Block-diagram of the Toekang Besi-group. Horizontal scale ± 1 : 600.000, vertical scale ± 1 : 300.000. The steep submarie slopes, that are formed of reef rock and reef detritus are indicated separately.

southwestern islands and submerged banks, the steep slopes of the latter towards the flat central plateau, all this is strongly in favour of HETZEL's

Fig. 1. Bathymetrical chart of the Toekang Besi-islands, scale 1 : 1.250.000. (From data collected by the SNELLIUS-expedition and the Hydrographical Survey).

theory, that the last movements were blockfaulting with deeper lying folding. This principle is the same as that proposed by MOLENGRAAFF for the youngest geological history of the outer Banda-arc.

We must now consider the mode of formation of the reef-islands, and attempt to decide, whether glacial control or upgrowth during slow sinking of the foundation, best explains the present configuration. All arguments point to the latter explanation.

1. The elevated islands are formed of soft, friable chalk and marl, covered by reef limestone of varying thicknesses. If low-level abrasion had been active in this group, we should expect to find at least some indication of abrasion in this soft material, but most of the islands are continued directly in submarine slopes that fall off steeply and uninterrupted to two or three hundred meters. If formerly islands existed where the atolls are now found and these had been abraded during the pleistocene, why do the present islands of soft material show such poor signs of pleistocene abrasion? Roendoema in the northeast is more exposed than the central atolls and no adequate reason can be given why it was hardly even cliffed, while the atoll-foundations were abraded over a breadth of 10 km.

2. For various other reasons the former existence of islands below the present atolls seems unlikely. The ridge below the oblong atolls lies many hundreds of meters below sea-level. It seems most unlikely, that abrupt elevations brought it above sea-level here and there. The central reefs are not situated on ridges, nor do they form rows parallel to the strike of the group. If the arbitrary position of the latter were to be explained by assuming volcanic cores, the strong abrasion of these hard cores would be even more difficult to explain than in the case of soft marls. No volcanic material is known from this group, sothat this supposition is altogether too improbable.

3. The great stability of the substratum since the beginning of the pleistocene, that must be postulated for the glacial control, is most unlikely. The elevated terraces and the folded and faulted seabottom prove the tectonic activity in this region up to a recent date and are in complete accordance with the movements necessary for DARWIN's theory to explain the atolls.

4. The most convincing argument in favour of DARWIN's theory is to be found in the submarine sections of the reefs and the elevated islands. The sections of our figure 2 are drawn to scale and show the *steepest* submarine slopes of the elevated islands and a number of *average* slopes of the atolls and of the lagoonless reefs. The difference is therefore even greater than appears from this figure. The fact, that the atolls and reefs are much steeper than the islands, and continue down to 500 or 600 m at least, in these precipitous slopes cannot be explained by the theory of glacial control, that assumes the atolls to be non-coralliferous below the depths of about 100 meters. A study of charts shows that fine waste-matter,

as produced by abrasion,, does not accumulate in slopes of nearly so acute a declivity. Even volcanoes are very seldom steeper than 25° below the surface.

There is one way in which these exceedingly steep slopes down to depths of 500—600 m can be explained, namely by the accumulation of the coarse detritus of growing reefs. Even this material is not able to accumulate at the angles of some of these sections. For these most extreme cases we must assume, that vertical upgrowth of coral reefs on a sinking substratum has taken place over several hundreds of meters of thickness.

Is was stated, that the small reefs belong to the same class as the atolls. The absence of a lagoon is easily explained by the small extent of the surface. The submarine sections are seen to be similar to those of the atolls. This confirms the view, that both structures were built in the same manner and that the difference between atolls and islands is not a mere coincidence. The comparative steepness of the upper portion of the island sections can be explained by talus-formation of reef detritus on the slopes of the islands during periods of slower elevation or of standstill. The northern islands must also have performed positive movements to form the barrier-reefs that occur here and there.

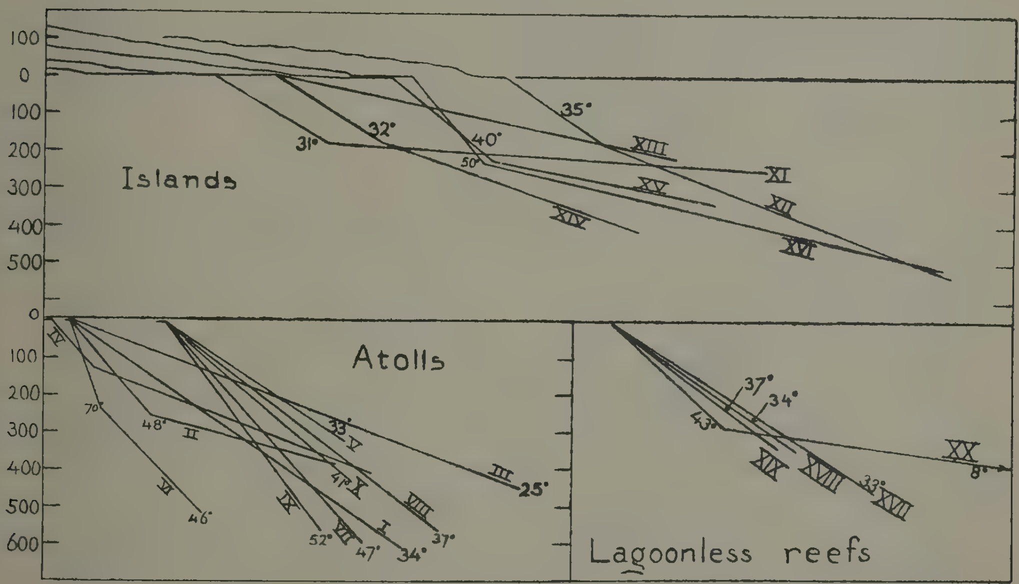


Fig. 2. Submarine sections of various formations in the Toekang Besi-group. Vertical scale and horizontal scale alike.

We are now able to outline the formation of the group:

Towards the close of the Tertiary epoch the present site of the Toekang Besi-islands was occupied by a slightly undulating plateau close below the surface of the sea. Round the highest points of the anticlines grew reefs. During the subsequent development of the region the general level

gradually sank several hundreds of meters. A number of reefs grew up and formed either oblong atolls on the initial anticlines or round atolls and small, solid reefs on the plateau, while other reefs were slowly but intermittently raised above sea-level. On the whole these movements were in the nature of block faulting, although the original subdivision in anticlinal ridges was more or less retained, so that folding, deeper down in the crust, probably continued. One of the reefs, Lintea-zuid, was tilted as ESCHER pointed out, and now consists of an atoll with an elevated rim on the side of the row of islands. The islands sometimes sank a small distance, giving rise to the formation of barrier reefs here and there.

We see, therefore, that ESCHER's theory of the origin of the group has been hardly altered. Tectonic influences produced the arrangement of the islands and atolls. The latter were formed by the subsidence of the substratum. The only additions to be made are, that the movements were principally faulting (HETZEL's suggestion), that the northeast anticline follows a different line, and that the reefs are situated on inconspicuous elevations of the sea bottom, which were later depressed by a general sinking of the plateau (RUTTEN's suggestion).

There are very few atolls, that betray the nature of the core on which the visible coral-structures grow. In the foregoing article I have attempted to show, that for the atolls in the Toekang Besi-group the organic nature can be proved of the atoll-structure down to depths of several hundreds of meters, perhaps even of one thousand meters.

BIBLIOGRAPHY.

1. B. G. ESCHER: Atollen in den Nederlandsch Oost-Indischen Archipel. Med. Encycl. Bur., Weltevreden, 22, 1920.
2. H. W. HETZEL: Over de geologie der Toekang Besi-eilanden. De Mijningenieur, 9, 1930, p. 51—53.
3. PH. H. KUENEN: Geologische problemen in verband met de Toekang Besi-eilanden. Tijdschr. Aardr. Gen., 1928, p. 236—247.
4. G. A. F. MOLENGRAAFF: Geologie. In: De Zeeën van Nederlandsch Oost-Indië, 1922, p. 272—357. Translation in: The coralreefs in the East Indian Archipelago, their distribution and mode of development. Proc. Fourth Pac. Sc. Congr. Java, 1929, Vol. 2, p. 55—99.
5. L. M. R. RUTTEN: Voordrachten over de geologie van Nederlandsch Oost-Indië, 1927, p. 635—649.

Geology. — *On the geophysical foundations of the Undation-theory.* By R. W. VAN BEMMELEN. (Communicated by Prof. G. A. F. MOLENGRAAFF).

(Communicated at the meeting of March 25, 1933).

In a previous paper (1933*b*) I considered the origin and the kind of the tectonic forces, and I argued magmatic differentiation to be able to give potentional energy to gravitation. After its publication the paper written by KUENEN (1932) came to my knowledge, in which some remarks on the Undation-theory are made. The following will merely treat about his objections concerning the primary tectonics. Those concerning the secundary tectonics I intend to discuss more detailed somewhere else.

Concerning the origin of the earth BERLAGE (1932) stated: "We have found such convincing evidence of a gaseous disc having constituted the original state of our planetary system, that I feel obliged to express doubt, whether the tidal theory of origin of this system will remain any longer a serious competitor besides some other theory, which follows the line of thought of Descartes, Kant and Laplace."

From the moment the earth condensed from a nebula to a redhot liquid sphere a continuous cooling down by radiation took place. It is improbable, that radioactivity would ever again cause a rising of the temperature in this phase of its development.

This redhot liquid sphere possessed at first relatively little consistency. Consequently the situation was extremely favourable for an arrangement of the material according to spec. gravity as the sphere was in the gravitational field created by the earth itself. According to GOLDSCHMIDT in this stage moreover a first physico-chemical separation of the material took place, which acted in a similar manner as the process taking place in a blastfurnace (metalliferous core, an oxysulfidic layer and a silicate mantle; the outmost 100 km of this mantle form the crust of the earth).

The calculations of ADAMS (1924), JEFFREYS (1927), GUTENBERG a.o. all point to the conclusion, that since the development of a crust (i.e. since the beginning of geological history) the outmost 100 km of the earth (the present crust) are chiefly concerned in this cooling.

Underneath the present crust the temperature can have dropped only slightly under its initial value.

This silicate mantle of the earth formed an extremely complex melt, which included during its condensation a great many volatile components. It is to be expected, that changes in P and T will cause in their turn changes in the physico-chemical equilibrium of this melt. By continuous cooling some of the components will attain their temperature of crystalizing,

and the volatile components become more effective in transfer of material. Therefore cooling caused the tendency of differentiating the original melt. These products of differentiation will partly possess a lower spec. grav., partly a higher one than the original material.

The problem to which depth the crust of the earth is of a crystaline nature, which problem is treated by DALY (1928), a.o., is only of secondary importance. From a theoretical point of view it is to be expected, that within material, crystalized at a great depth under high pressure, still large amounts of volatile components remain included. On pressure relief the material will become active on account of those volatile components, just as well as glassy material. This pressure relief causes then a gasdiffusion and selective transfer of material, resulting in splitting up in products of differentiation. These products will be in physico-chemical equilibrium with the new T and P conditions.

The real foundation of the Undation-theory is this tendency to differentiation of the outmost 100 km of the original silicate mantle as a consequence of its cooling, which throws the material out of its physico-chemical equilibrium. This furnishes an enormous source of energy, until now not at all or insufficiently taken into account, which enables us to explain for the greater part, if not completely, the endogenic (tectonic and volcanic) phenomena.

Another problem is in what way this process of differentiation takes place. At first I chiefly thought of fractional crystalisation as claimed by BOWEN (1928).

KUENEN considers this as a bold assumption and says that to his opinion "differentiation in the present earth's crust as a whole must be considered impossible, except in limited masses in small depths, where liquids can exist in the form we know them."

I never intended to say that the crystalls will sink all by themselves for some tens of kilometers. A differentiation of a layer thick some tens of kilometers must always take place at a number of removes. When at a certain depth the conditions for differentiation are favorable, products will accumulate, which partly possess a lower spec. grav. (sial or sal) and partly a higher one (sima) than the original material (called: salsima). This causes a deviation of the hydrostatic equilibrium.

Now the second leading principle of the Undation-theory is, that in the zone of rockflowage, in which the consistency of the rocks is nearly nihil, no everlasting deviations from the hydrostatic equilibrium can exist [1]).

[1]) GUTENBERG (1930*a*) pointed out, that: "Forces the importance of which seems to have been overlooked are those caused by *the deviation of the earth's crust from hydrostatic pressure*". In his theory of continental flow, which is based on this principle, GUTENBERG assumes a primary salic block, which under the action of the hydrostatic forces began to drift apart. This assumption however is not in agreement with the development of the continents as taught by geologic history, which points more to a steady growing both in thickness as well as in extension of the sialic shell during the geologic history.

In case rather important accumulations of differentiation-products have been formed, those, which are lighter than the surrounding original material will rise, while the heavier ones will sink down.

Their place will be taken in by fresh not yet differentiated material, so that the process of differentiation can continue. In this way at some depth the accumulation of products of differentiation increases. At first these local displacements of material in the underground do not yet cause oscillations at the surface. As such a process of differentiation in the underground does not change at all the total mass of the differentiating column, no gravitation anomalies are caused by it.

Finally the accumulation of products of differentiation increases to such an amount, that for the restoration of the hydrostatic equilibrium the adjoining columns get implicated in the process of massdisplacements. From this moment, these massdisplacements in the underground will produce primary tectonic movements at the surface and temporary deviations of isostasy, because in this stage the total volume and weight of the columns will be changed.

In hydrodynamical respect these displacements of the lighter sal and the heavier sima are two processes which are independent of each other. The (very small) speed of these displacements depends upon the difference of hydrostatic pressure between the columns lying next to each other and also upon the absolute value of this hydrostatic pressure; the internal friction being influenced by the latter. The difference of spec. gravity of these products of differentiation in relation to the original material, as well as the absolute value of the hydrostatic pressure are different for the sal in the upper part of the zone of rockflowage and the sima in the lower part of it. This causes, that the speed, in which the hydrostatic restoration-movements for the salic and the simatic material take place, will be different. This speeddifference of hydrostatic adjustment of the salic and the simatic material will in its turn give rise to temporary deviations of isostasy.

In my previous publications is explained how these massdisplacements in the zone of rockflowage will cause undations at the surface, and how under favorable conditions a sideways shift of these undations may occur (undation-cycles). In respect to the remarks of KUENEN I only wish to point out, that for such a process of differentiation it is not necessary at all, that the viscosity of the parental material is so low that transversal seismic waves will be extinguished.

FENNER (1926, 1929) drew the attention to the serious neglect of the influence of gasses in the process of differentiation by BOWEN. The volatile components cause by escaping a selective transfer of material from lower to upper levels. Such a process will also result in a differentiation of the parental material, so that this density stratification does not need to depend merely on crystal-fractionating as put forth by BOWEN.

Merely the fact, that any process of differentiation will cause a density

stratification and in its turn local deviations of hydrostatic pressure in the zone of rockflowage is of importance for the Undation-theory. In the first place this gives rise to primary and secondary tectogenesis at the surface and secondly continuously new parental material is set to differentiate. This process will continue until by complete cooling the potential power for differentiating of the original silicate melt has been consumed and a perfect adjustment to the hydrostatic equilibrium of these differentiation-products has been attained.

In 1933*b* I drew attention to the fact, that the position of the surfaces of discontinuity in the outer shell is still uncertain. The same holds good for the composition and spec. grav. of the parental material (salsima) and the products of differentiation (sal and sima).

To have some basis for drawing the profiles (1932*b*, Fig. 13) I accepted the depths of the seismic surfaces of discontinuity for continental areas as given by JEFFREYS. JEFFREYS's figures are considerably lower than those generally accepted by other seismologists. MOHOROVIČIĆ (1927) for instance assumes that underneath normal continents the sialic layer has a thickness of 40 km, underwhich follow 20 km of sialma (40—60 km depth) and then 60 km of sifema (60—120 km depth) [1]).

For the Pacific I assumed the salsimatic surface layer to have a thickness of 40 km on account of the calculations of BYERLY (1930). GUTENBERG (1930*b*) however pointed out that BYERLY presupposed (in the interpretation of the dispersity of the Love-waves) a stratification of the underground. This dispersion can also be explained, according to GUTENBERG, by a gradual increase of the wavevelocities with the depth. As a consequence of this my assumption for the thickness of the salsima layer under the Pacific has lost its ground. The assumption that the hydrostatic restorationmovements will be in force only to a depth of 60 km was merely made on account of the consideration, that with greater depth the hydrostatic pressure and also the internal friction increase, which will impede the restorationmovements. The observations of WADATI (1928) and VISSER (1929) on earthquakes with foci at depths of about 400 km make it probable, that this limit is situated much deeper.

According to the general considerations made in the first part of this paper it is probable that the tendency for differentiation of the original silicatemantle exists in the outer 100 km, which since the forming of the earth have cooled most. It is however possible, that deeper parts of this original melt will be taken up in the differentiationprocess too, because they are pressed to higher situated zones as a consequence of the sinking down of the heavier basic differentiation-products into this deeper situated parental material. To what extent such interchanges take place depends

---

1) Sial = granite, granodiorite and some syenite.
Sialma = gabbro and diorite.
Sifema = pyroxenite and peridotite.

upon the difference of spec. grav. between both and the possibility of the material to move at those depths. It is also possible that crystallographic-chemical changes into minerals with smaller molecular volumes occur, so that for lack of data little can be said concerning these replacements at great depth.

The isostatic compensation of high mountainranges as Himalaya points to a very considerable sialic mass underneath them. In trying to explain this large sialic mass by differentiation of the parental material in situ it is necessary to accept the differentiation to have taken place here to a very great depth. *Now one of the principal conclusions of the Undationtheory is, that geosynclinal subsidence will stimulate differentiation of the parental material,* because these primary tectonical movements correspond to mass displacements in the depth, which cause the parental material to be relieved from an enormous pressure whilst a relative heating up takes place as the result of the bending upward of the geoisothermal surfaces (1932*a*, p. 396; 1932*b*, p. 124—126; 1933*a*). The Himalayan-geosyncline has been according to GRABAU the longest living geosyncline we know. From the silurian up to the oligocene a nearly uninterrupted sedimentation has taken place. This longlasting primary tectonical subsidence might point to the fact that here during a very long period and to a very great depth this process of differentiation really has taken place. During the miocene finally the hydrostatic upheaval of the salic products has been brought about on a large scale, producing the highest mountainrange we know.

KUENEN is of opinion that the problem of radioactive heatdevelopment has not been sufficiently taken into account. In 1932*b*, § 5, p. 108—109, was already pointed out, that the assumptions concerning the distribution of the radioactive elements in the crust of the earth are rather speculative. See in this respect also BAILEY WILLIS (1932).

Radioactivity no doubt is a source of energy that must be taken into account. But only its qualitative influence can be estimated. This heat-development increases the mobility of the material in the zone of rock-flowage and it therefore promotes igneous activity and massdisplacements. As such it is a stimulating factor for the magmatic and dynamic processes discussed in the Undation-theory.

KUENEN further remarks, that a clearer exposition is required, why the apparently continuous process of magmatic differentiation could result in a rhythmical pulsation. In respect to this remark see my discussions in 1932*a* and 1932*b*, § 9, p. 147—148, to which the following may be added. In nature we know a great many periodical processes which possess a rather constant amplitude but a less constant period. The period of such processes is determined by the time needed for heating up, for loading up, for diffusion, for filling, a.s.o.: this time is called the time of relaxation. These vibrations, called *relaxation-vibrations,* originate when by some source of

energy a typical aperiodic phenomenon is periodically repeated (definition given by VAN DER POL in 1930).

In our case the source of energy is the magmatic differentiation. The periodically repeated phenomenon the undation. Some accumulation of differentiationproducts is necessary, before the adjoining columns are taken up in the process of massdisplacements and the undatory uplift takes place. The time, required by magmatic differentiation to produce this accumulation is the relaxation-time. The dimensions of the accumulation of the salic products of differentiation and the difference of spec. grav. between them and the surrounding material determines the dimensions of the resulting undation.

In the future it will perhaps be possible to estimate the periodicity of the undations (in general the periodicity of orogenesis) with the differential equation for relaxation-vibrations given by VAN DER POL (1926):

$$v'' - E(1-v^2)\, v' + v = 0$$

v is the speed of the primary tectogenesis.

E is a positive quantity $\left(\frac{log.\ increment}{\pi} > 1\right)$.

Bandoeng, Febr. 1933.

REFERENCES.

1. ADAMS, L. H. 1924: Temperatures at moderate depths within the earth. J. of Wash. Ac. of Sc., **14**, 20, 1924.
2. BEMMELEN, R. W. v. 1932*a*: Über die möglichen Ursachen der Undationen der Erdkruste. Proc. Kon. Ac. v. Wet. Amsterdam, **35**, 3, 1932.
3. ——— 1932*b*: De Undatietheorie (with english summary). Nat. Tijdschr. v. Ned. Indië, **92**, 1, 1932.
4. ——— 1932*c*: Nadere toelichting der Undatietheorie (with english summary). Ibidem, **92**, 2, 1932.
5. ——— 1933*a*: Die Undationstheorie und ihre Anwendung auf die Mittelatlantische Schwelle. Zeitschrift d. Deutschen Geol. Ges., 1933 (in print).
6. ——— 1933*b*: Über das Kräfteproblem in der Tektonik. Proc. Kon. Ac. v. Wet. Amsterdam, Febr. 1933.
7. BERLAGE, H. P. 1932: On the structure and internal motion of the gaseous disc constituting the original state of the planetary system. Proc. Kon. Ac. v. Wet. Amsterdam, **35**, 4, 1932.
8. BOWEN, N. L. 1928: The evolution of igneous rocks. Princeton, 1928.
9. BYERLY, P. 1930: The dispersion of seismic waves of the Love-type and the thickness of the surfacelayer of the earth under the pacific. Gerl. Beitr. z. Geoph., **26**, 1930, p. 27.
10. DALY, R. A. 1928: The outer shells of the earth. Am. J. of Sc., **15**, Febr., 1928.
11. FENNER, C. N. 1926: The Katmai magmatic province. J. of Geol., **34**, 7, 1926.
12. ——— 1929: The crystallisation of basalts. Am. J. of Sc., **18**, Sept. 1929.
13. GOLDSCHMIDT, V. M. 1922: Über die Massenverteilung im Erdinneren, verglichen mit der Struktur gewisser Meteoriten. Die Naturwissenschaften, **10**, p. 918—920, 1922.

14. GUTENBERG, B. 1930*a*: Hypothesis on the development of the earth. J. of Wash. Ac. of Sc., **20**, 2, 1930.
15. ——— 1930*b*: Der Aufbau des Untergrundes im Pazifischen Ozean. Gerl. Beitr. z. Geoph., **26**, 2, 1930.
16. JEFFREYS, H. 1927: On the earth's thermal history and some related phenomena. Gerl. Beitr. z. Geoph., **18**, 1, 1927.
17. KUENEN, PH. H. 1932: Remarks on the undationtheory of VAN BEMMELEN. Proc. Kon. Ac. v. Wet. Amsterdam, **35**, 9, 1932.
18. MOHOROVIČIĆ, S. 1927: Über Nahbeben und über die Konstitution des Erd- und Mondinnern. Gerl. Beitr. z. Geoph., **17**, p. 180—231, 1927.
19. VAN DER POL, BALTH. 1926: Over relaxatietrillingen. Physica, **6**, p. 154, 1926.
20. ——— 1930: Periodische verschijnselen in de Natuur. Diligentia, 1930.
21. VISSER, S. W. 1929: On the evidence of very deep earthquake-foci near West-Java. Proc. IVth Pac. Sc. Congress, Vol. IIA, p. 229, 1929.
22. WADATI, K. 1928: Shallow and deep earthquakes. Geoph. Mag. Tokyo, **1**, 4, p. 162, 1928.
23. WILLIS, BAILEY 1932: Radioactivity and theorizing. Am. J. of Sc., **23**, p. 193—226, 1932.

Biology. — *Sex-ratio variability and the problems of reproduction among lac-insects.* By S. MAHDIHASSAN. (From the Zoological Institute, Giessen. Direktor Prof. W. J. SCHMIDT.) (Communicated by Prof. H. F. NIERSTRASZ).

(Communicated at the meeting of March 25, 1933).

From the standpoint of plant-pathology scale insects may be divided into two classes, with and without parthenogenesis. A typical coccid is imagined as unisexual where each individual born is able to reproduce its kind and the species therefore capable of multiplying twice as fast as when males were to be in equal numbers. In this case it is easy to assume the wind as a simple distributing agent. However where pathenogenesis does not occur, it has to be understood, that a male and a female larva are transported together or in some way come to lie near each other which would make the wind an improbable means of distribution. The second hypothesis is that two or more larvae are transported by a fly and that both sexes are represented; this is not difficult to grant if it is known that the sex-ratio approaches equality. The chances of sexual reproduction, however, would tend to be reduced as the sex-ratio would deviate from it. The third possibility is that the wind scatters the individual larvae wide apart, for the wind cannot do otherwise, but that the adult male seeks out the female through its sense of smell as is known in other insects. It is proposed to examine these fundamental questions with particular reference to the lac producing coccid, *Lakshadia communis*.

CARTER (1) was the first to make the suggestion that the lac insect, like

the aphid, may multiply parthenogenetically but the idea was entirely forgotten until I found a new form of the adult female lac cell, resembling a crown, which I believed to be a parthenogenetic insect (5). Later observations made me change this view (9). The verdict of E. SHRADER is further quoted in favour of sexual reproduction among coccids: „I have been unable to find any evidence that previous investigators have considered the possibility that parthenogenesis may occur... despite the fact that most entomologists concerned with coccidae have strongly suspected that parthenogenesis may play an important role in the reproduction of the group" (14).

In the literature on coccids the natural means of distribution is very casually touched or entirely omitted. The problem gains an extra importance where insects were presumed to be parthenogenetic but observations could not substantiate it. The following considerations exclude the wind as a distributing agency of the lac insect. Most coccids leave their dead skins attached to the twigs so that the old range of infection can also be ascertained. This is particularly easy with lac insects which form visible encrustations and the dead colonies may be collected and sold as stick-lac. In Bangalore *L. communis* thrives on some very common trees: *Ficus mysorensis* is a regular avenue tree; *Anona squamosa* yields edible fruits and is also very plentiful, encrustations of lac are thus easily noticed on it when fruits are gathered; *Guazuma tomentosa* is, likewise, a frequent avenue tree. When lac is collected from these trees and the localities plotted on a map no kind of order is found; infection is not found on two or three neighbouring trees but spontaneously on trees scattered far and wide. With wind as a distributing agency it was expected the trees infected would show the direction in which it usually blows during the season of larval swarming. Briefly, all observations point to prove that the young insects are transported by some kind of fly, a theory first applied to the biology of the lac insect by the French lac-expert, M. HAUTEFEUILLE (2).

When we are prepared to consider the possibility of a fly carrying two or more larvae we are confronted with the rare chances of the sexes being different. In Bangalore there are two life cycles of *L. communis*. The mother insects growing during the rainy season produce offspring in October but the larvae develop, one and all, into winged males. The non-monsoon generation consists normally of scattered crown shaped cells which deliver larvae in May; the progeny consists of about 11 females to 1 male which is always wingless. In the first case the problem is not how two larvae are carried but how the lac insect can ever multiply in the entire absence of females. The only conclusion is that the labile sex-complex changes into a bisexual nature and the early stages of the hermaphrodite have been already illustrated (9). The crown shaped cell is nothing else than the larva of the winged male transformed to display the functions of both the sexes. The fine cytological work of Mrs.

SCHRADER explains a similar case in another coccid (13). When the ratio is 11 females to 1 male there are likewise 5 chances of two larvae of the same sex being carried to only 1 of their being different. My observations show that the larvae are transported singly for crown shaped cells are found one on each twig; and very likely only one or two larvae from each swarm are thus carried to other trees.

The last possibility to be explained is that the sexes have been scattered wide apart but that the adult male has the power of seeking out its female. We have to estimate in this case the powers of flight on the part of the male and the average distance separating the sexes. The solution is very simplified by the consideration that when the males are winged there are no normal females anywhere and when the latter occur the male is wingless and has to cover 11 females in the same colony before it can seek others at a distance.

It is evident the importance attached to the problems of reproduction arise principally from sex-ratio determinations which in itself implies identification of the sexes in the earliest larval stage. It is sufficient to find the sex-ratio of the larvae as they swarm out of the mother cells; those which have died within, without seeing the light of the day, would never have been otherwise distributed. Likewise such colonies where deaths have occurred cannot give the necessary sex-ratio, for it has been actually found that mortality is relatively higher among the males.

Attempt to determine the sex of young lac and other scale insects was made by IMMS and CHATTERJEE without success (3). No less an authority on coccids than Šulc, likewise, mentions: „Erste Larvae. Sie weist keine ausseren Geschlechtsunterschiede aus" (15). Nevertheless a long paper, with 26 Figures differentiating the sexes in the first stage accompanied by two macrophotographs of young colonies has been already published where the sex ratios have also been determined (6). The larvae shortly before and after the first moult have been further illustrated (8); the first stage larvae of the female, the winged and wingless male are again figured in another communication (10). The ventral views of the male and female crawling larvae have also been illustrated (7). These communications are mentioned to remove all doubts regarding the probability of early sex-ratio determinations.

By brood lac is meant encrustation of lac with living insects about to deliver swarms of young ones and as such used for propagation. In Bangalore this is obtained only at the end of the monsoons; the pieces are invariably small as previously illustrated (12) and contain each a crown shaped cell which is dry, indicating it as the mother cell which gave rise to the offspring forming the chunk of lac. Now these insects growing during the rainy season give rise only to winged males as has been already mentioned. In the second larval stage they form long cells differentiating themselves from those of the female and the wingless male, all of which have been previously figured (11). Since the lac insects have been studied

by a number of famous scientists and no one has recorded an excess of males much less their exclusive preponderence Fig. 1, Pl. 1 is reproduced in its support. The specimen belongs to *L. communis* growing on *Butea frondosa*, Lingal, Hyderabad State, July 1919 and the picture shows the insects magnified about 2 diameters. It represents the top portion of a long, vertically growing twig, encrusted with cells of winged male lac insects. The encrustation, not the twig, shows a gradual thickening towards the top being the direction in which all insects have been facing. The settlement has been so thick that the anterior portions of the cells are hidden and only their posterior regions are visible. The male leaves its cell through the opercular opening, marked O with an arrow in white ink, this opening is covered before its exit by the opercular lid, marked L with an arrow in black ink, Fig. 1. This is mentioned to indicate the direction in which the cells have been fixed as is evident from the photograph. With the encrustation in hand it would be easy to convince the cells belong to winged males, the photograph, so far described, however, only shows that the cells, which are individually indistinct, belong to males. The larvae of winged males, however, as distinct from those of the wingless male and the female, show high phototropism ; they thirst, so to speak, for light and crowd themselves competing with one another for front seats. As we get towards the top we are nearing the source of light and crowding is proportionally heavier. The twig freed from the encrustation would naturally show the basal portion thicker than the top end. I must mention, upto now, I have not seen a better example of phototropism among lac insects than shown in Fig. 1, so that the larvae of winged males are thereby easily distinguished.

As larvae are abundant only during October and scarce during May it follows distribution from tree to tree occurs solely at the end of the rainy season. The mother insects which form the chunks of broodlac also give rise to excreta containing both glucose and fructose and on this honey thrive bacteria which form esters of butyric acid and attract flies feeding on slimy products ultimately enabling the young ones to be carried to other trees. The larva belonging to the winged male must, however, become bisexual for which it is necessary each to be singly distributed. On the same tree crowded along with other winged males this transformation never occurs. That the crown shaped cells are derived from the winged males is easily proved by the fact that during the other season, when both females and wingless males are found, the bisexual forms are never found either in the same colony or scattered singly. Briefly then, from October till May the generation is carried by the crownshaped cell while its contemporaries have all died as bachelors. The transformation is accompanied by an enlargement in its size, the crown shaped cell is almost twice the size of the normal female cell. The latter delivers as a rule some 700 larvae ; the former has been observed to give once 1435 females and 114 males, the counting was done in the second larval stage

S. MAHDIHASSAN: SEX-RATIO VARIABILITY AND THE PROBLEMS OF REPRODUCTION AMONG LAC-INSECTS.

PLATE I.

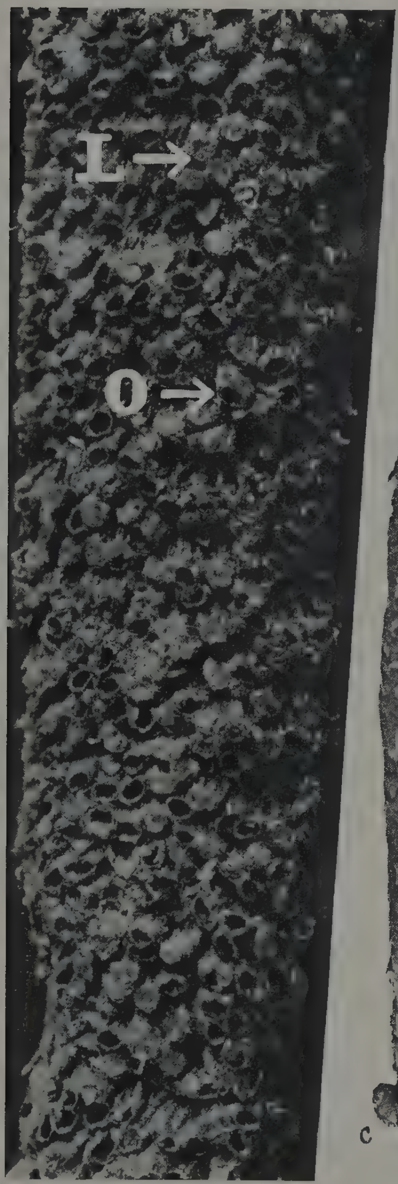


Fig. 1

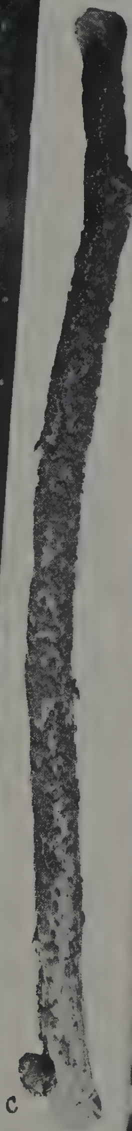


Fig. 2

Fig. 3

Proceedings Royal Acad. Amsterdam, Vol. XXXVI, 1933.

where deaths had been noticed, hence the actual number of larvae born must have been higher. The crown shaped cell thus functions as more than two individuals. In order to bring 1500 larvae into the world the cooperation of at least two females and one male would be necessary besides the natural difficulty of transporting two or more different larvae at the same time.

Fig. 2, Pl. 1, shows a crownshaped mother cell, marked C, showing half the progeny, of some 1500 larvae in the second larval stage, on one side of the twig. It represents 10 : 9 natural size and was a vertical twig hanging downwards from a horizontal branch of *Nephelium Litchi*, Botanical Gardens, Bangalore, June 1923. The crownshaped cell was nearer the joint with a horizontal branch so that the larval settlement has occurred most at the other end of the twig. Light falling from above the end with the crown shaped cell had more share of it than its growing end and yet the larvae have settled in the direction away from light. The young lac insects exhibit geotropism so that on a vertical twig growing upwards its basal end, which is nearer the earth, would be the more thickly populated; the fully developed encrustation would therefore show a gradual tapering shape thinning towards the top as illustated in Figs. 4 and 6 (9). A vertical twig growing downwards has its basal portion attached to a horizontal branch and away from the earth and larvae reacting to geotropism would now prefer the growing end of the twig; the fully grown encrustation would appear club-shaped thickening as we proceed away from the base of the twig although this in itself is thinner at the growing end. Such an illustration would be reproduced in another communication on photo- and geotropism among lac insects. It interests us here to observe that apart from the fact that the crown shaped cell gives rise invariably to females and wingless males the picture reproduced in Fig. 2, Pl. 1, is characteristic of a generation where both the sexes are present. Fig. 1 was selected to show an extreme case of phototropism in order to distinguish, thereby, the winged males, Fig. 2 likewise represents, an extreme case of geotropism indicating the presence of both the sexes among the larvae. That females predominate in such a generation is not as yet evident from Fig. 2.

As already mentioned the total progeny arising from all the crown shaped cells is very small on account of the scarcity of the mother cells. These are really so scarce that during the eight years in Bangalore I could observe only a few cells where the larvae were swarming or had swarmed a month ago. They are however easily collected along with the encrustations formed at the end of the monsoon season, when the former are empty. During 1923 the monsoons in Bangalore failed and the temperature went upto 103 F., a record for the previous thirty years. Under these circumstances chunks of brood lac obtained during November, for the delayed monsoons had lengthened the time of maturation, gave rise to abnormal sex ratios the following generation. As usual lac was infected on different trees and records kept. Many of them showed a mixed generation; the

winged males predominated, while the wingless males and females were also present. In May 1924 brood lac of the non-monsoon season was obtained for the first time in Bangalore and the offsprings contained the same sex ratio as delivered by the crown shaped cells. Mother-insects growing during the dry season, normal or otherwise, give progeny with the same sex ratio. Fig. 3, Pl. 1, is specially reproduced for it shows hardly any dead larvae in the colony, as the reader can verify for himself. Five cells of wingless males are numbered, Nos. 1 and 5 in white and the rest in black ink; all the rest belong to the female. A portion of the colony with the male larval cell, No. 3, is further enlarged in Fig. 4, Pl. 2. The male in the second larval stage is distinctly larger than the female of the same age, contrary to the finding of IMMS and CHATTERJEE. The female cell which has just moulted for the second time with its moulted skin marked s., is seen in the female cell marked III; most female cells are in the early third stage. A couple of cells are still in the second stage, one of which is marked II. The picture has been retouched to bring out the outline of the cells.

In Fig. 3, Pl. 1, it would be noticed that all the male cells point in one direction. The insects had settled all around an upwards growing twig, of *Guazuma tomentosa* yet the male larvae do not show eagerness to press forwards and to acquire illuminated seats. It must be remarked, however, they, one and all, point in the direction of light. The numerous female cells on the contrary show two directions of settlement, some facing the source of light while others the very reverse. Only female larvae show this poverty of phototropism. Fortunately this phenomenon has been unwittingly illustrated by IMMS and CHATTERJEE in their Fig. 3, Pl. 2, where 8 female cells are figured, 5 facing one direction and 3 the very contrary. This is said to suggest that when larval settlement shows the cells to have fixed themselves in different directions it is a rough indication of females predominating. Although Fig. 4, Pl. 2, represents only a small number of cells still it also bears out this phenomenon which is better seen in Fig. 3, Pl. 1. It is thus clear that the generation issuing from mothers growing during the non-monsoon season contains some 11 females to 1 male as Fig. 2 would verify and this contrasts itself with Fig. 1 where all are males.

CARTER, of all previous writers, alone gives a sex-ratio finding. The generation issuing at the end of the monsoons has an equal number of males and they are all winged. The other generation has two females to one male which here is wingless. He was thus the first to establish an alternation of generation among males towards which WEBER rightly draws the attention of his reader, saying, that the lac insect, "zeigt sogar einen mit Flügelpolymorphismus verbundenen Generationswechsel. In der ersten Generation dieser Lackschildlaus gibt es nach IMMS und CHATTERJEE geflügelte und ungeflügelte Männchen, die zweite Generation hat nur ungeflügelte. Dieser regelmässige Wechsel erinnert etwas an die bei den Aphidenen vorliegenden Verhältnisse, wenn wir auch nicht sicher sagen

PLATE II.

Fig 4

können ob diese Ähnlichkeit nicht nur oberflächlich ist, ob die Flügelreduktion bei den Cocciden nicht in Wirklichkeit nach den bei den pterygopolymorphen Wanzen gültigen Regeln eintritt" (16). Where females occur wingless males occur; CARTER casually mentions this fact and its confirmation has been the particular contribution of IMMS and CHATTERJEE. How the sex ratios depend on the weather was not observed before. Upto now the two life-cycles have been called the winter and the summer crops. To show the great influence of moisture on the life of the lac insects I have introduced the terms monsoon and non-monsoon crops. Similarly the existence of crown shaped cells, the occurrence of pure colonies of winged males, as well as the great preponderance of females at the other season, 11 or more females to 1 male, has been overlooked.

The important fact to be borne is the influence of water on sex ratios of lac insects. The generation containing 11 females to 1 male may be imagined as not conducive for sexual reproduction. It may be supposed that one male is not enough for all the 11 females to be fertilised. But among their progeny year after year no other result has been obtained than pure colonies of winged males; at least some females could be imagined as being fertilised so that the coming generation ought to be mixed. The usual ratio between the sexes under drought conditions of 1923 gave a mixed generation the following season. The importance of the climatic factors as different from that of sex-ratio is thereby very apparent.

The supply of water increases the growth of mother insects, the size is greater, the number of larvae born is higher, also mortality is less; but inspite of all these favourable conditions the next generation is predisposed to be males. Among aphids the reverse is known, viz. conditions of plant growth are helpful for the female sex determination. Among lac insects it has been definitely established that conditions of growth and female sex determination are opposite to each other.

In 1919, I published a paper on the cultivation of lac from a physiological standpoint (4) giving results of observations concentrated mainly on the plants infected by the lac insects. No account could then be taken either of the different species of lac insects or of sex ratio variability. Yet the conclusion was arrived at that the lac parasite plays a very passive role in the attack; the trees have first to prepare slimy products and then invite the insect to suck out the undesirable byproducts. Here the same conclusion is independantly arrived at: the tree in vigourous growth with good supply of water is so able to change the sex ratio of the lac insects that not one female is left for the species to continue upon it in the future. A better illustration of the passivity of the parasite cannot be imagined. Such sudden disappearances of other scale insects are also known and attributed to the weather, or in the words of MITCHEL to, "a mysterious something, an occult quality of the atmosphere". It is proposed to show elsewhere how these insect epidemics may sometimes be

explained by sex-ratio variability in favour of the female and their spontaneous natural regulation by a preponderance of males.

Grateful acknowledgement is made of the kind interest of Dr. h. c. NAVAB SIR HYDER NAVAZ JUNG, Kt., L. L. D., Finance Minister, Hyderabad State, but for whose encouragement the work could never have been carried to the present stage.

EXPLANATION OF THE FIGURES.

Pl. 1, Fig. 1. Colonies of cells formed solely by the larvae of winged males. The encrustation is gradually thickening towards the top indicating more intense colonisation due to phototropism.

Fig. 2. A hermaphrodite crownshaped cell, C., with its offspring consisting of both sexes; half the population lies on the other side of the twig and is hidden from view.

Fig. 3. Mother insects growing during the non-rainy season give rise to more females than males, the latter being wingless. Male cells are numbered and face one direction. The remainder are females which have settled in mixed directions.

Pl. 2, Fig. 4. A portion of Fig. 3 enlarged with the wingless male cell N⁰. 3 which has not yet moulted for the second time. The female cells are mostly in early 3rd larval stage e.g. the cell marked III and the second moult skin, S.; Cell II has not yet moulted for the second time.

REFERENCES.

1. H. J. CARTER (1861). On the natural history of the lac insect. *Ann. and Mag. Nat. Hist.* 3rd Ser., Vol. **7**, p. 363.
2. L. HAUTEFEUILLE (1924). Report on Lac and its Industrial Treatment English-Translation. Govt. Press Hyderabad Dn. Role of flies mentioned on pp. 35 and 60.
3. IMMS and CHATTERJEE (1915). On the structure and biology of *T. lacca*. *Indian For. Mem.*, Vol. **3**, Pt. 1.
4. S. MAHDIHASSAN (1919). Cultivation of lac from a physiological standpoint. *Quart. J. Mysore For. Assoc.* N⁰. 3, pp. 197—204. A French translation in *Bulletin économique de l'Indochine*. Hanoi. 1924, N⁰. 164.
5. ———— (1923). Classification of lac insects from a physiological standpoint. *J. Sci. Assoc. Maharaja's Col.*, Vizianagram, Vol. **1**, on pp. 65 and 66 Crownshaped cell is mentioned.
6. ———— (1926). Early recognition of sex among lac insects. *J. Indian Inst. Sci.*, Bangalore, Vol. 9A., Pt. 1.
7. ———— (1929). The first stage larva of lac insects. *Btn. Ent. Res.*, London, Vol. **20**, p. 241.
8. ———— (1930). The first ecdysis of the lac insect. *Deutsch. Entom. Zeit.*, p. 527.
9. ———— (1930). The probable occurrence of sex-reversal among lac insects: *Zeit. f. Ang. Entom.*, Vol. **16**, p. 527.
10. ———— (1931). Early sexual and other dimorphisms among lac insects. *Biologia Generalis*, Vol. **7**, p. 517.
11. ———— (1931). The metamorphosis of the male lac insect. *Casopis: Tschekisch. Entom. Zeit.*, p. 5.
12. ———— (1932). The symbiote of *Lakshadia communis*. *Arch. f. Prot.*, Vol. **78**, p. 514.

13. Mrs. S. HUGHES—SCHRADER (1925). Cytology of Hermaphroditism in *Icerya purchasi*. *Zeit. Zellfor. Mik. Anat.*, Vol. 2, p. 264.
14. F. SCHRADER (1929). Experimental and Cytological Investigations of the Life-cycle of *Gossyparia spuria* (*Coccidae*) and their Bearing on the Problem of Haploidy in Males. *Zeit. wiss. Zool.*, Vol. 134, p. 149.
15. K. ŠULC (1932). Die tschechoslowakischen *Lecanium*-Arten. *Acta Soc. Sci. Natur. Moravicae*. Brno. Vol. 7, Pt. 5, p. 108.
16. H. WEBER (1930). Biologie der Hemipteren. Springer, Berlin, p. 405.

Anatomy. — *Ueber die basale Opticuswurzel und die caudalen Verbindungen der Commissura transversa Gudden der Vögel.* Von EUGEN FREY (Zürich). (Communicated by Prof. C. U. ARIËNS KAPPERS.)

(Communicated at the meeting of March 25, 1933).

Für diese Untersuchung wurden die folgenden Präparate des Niederländischen Zentralinstituts für Hirnforschung in Amsterdam benutzt: Markscheidenserien von Passer domesticus, Melopsittacus undulatus, Columbia domestica, Cacatua roseicapilla, und andere. Vereinzelt wurden auch einige nach Nissl, Heidenhain oder Cox gefärbte Serien untersucht. Ausser den Transversalserien wurden auch in einigen Fällen Sagital- und Horizontalschnittserien verwendet.

Ein Frontalschnitt durch den Thalamus eines Melopsittacusgehirns, etwas frontal von der Commissura anterior, gibt ein sehr übersichtliches Bild aller Decussationssysteme an der Basis des Thalamus. Ganz dorsal ist die Commissura supra-optica dorsalis (Commissura Meynert), die ein ziemlich mächtiges Bündel von relativ dicken Fasern darstellt, zu sehen. Da sie eine ausgesprochene caudo-dorsale Richtung einnimmt, ist nur ein relativ kurzer Abschnitt derselben am Frontalschnitt zu sehen. Unmittelbar ventral von der Commissura Meynert liegt ein viel schmaleres Bündel feiner schwach myelinisierter Fasern, das nicht so stark caudalwärts strebt und deswegen am Schnitt ein länglicheres Gebilde als die Commissura supra-optica dorsalis darstellt. Ventral von der Commissura transversa, nach einem deutlichen Zwischenraum, tritt die mächtige Kreuzung der Optici hervor.

Die Betrachtung der weiteren Schnitte caudalwärts zeigt, dass die Entfernung zwischen der Commissura transversa und dem Chiasma noch deutlicher wird. Am Schnitt durch die Ebene der Commissura anterior ist die Commissura Meynert bereits aus der Mitte verschwunden und ist beiderseits in der mittleren Höhe des Thalamus, sich fächerförmig ausbreitend und in der Richtung des Tractus thalamo-frontalis und des Tractus occipito-mesencephalicus strebend und gleichzeitig aufsplitternd, zu sehen. Die Commissura transversa behält ihre zentrale Lage bei deutlicher Entfernung

vom Chiasma und ihre Richtung mit der leichten Neigung zum Aufsteigen. An ihrem lateralsten Abschnitt kommt sie unmittelbar medial und etwas caudal vom Ganglion geniculatum laterale zu liegen und zeigt dort eine allmähliche Tendenz zur Aufsplitterung. Diese Verhältnisse sind sehr deutlich aus der Abbildung 5 ersichtlich (com. ant. = Commissura anterior, co. M. = Commissura Meynert, co. tr. G. = Commissura transversa Gudden). Zwei grosse Kerne beherrschen hier das Bild: es sind das Ganglion geniculatum laterale (g. g. l.) und der Nucleus rotundus (nu. rot.). Der erste Kern ist oberhalb des Tractus länglich gestreckt gelegen. Er sendet eine nach oben lateralwärts ziehende Bahn, die zwischen den Nucleus rotundus und den Tractus zu liegen kommt, sich weiter oben zur thalamo-frontalen Faserung gesellt und von mir Tractus geniculo-frontalis genannt wird (tr. gen. f.).

Einige Schnitte weiter caudalwärts sieht man den hinteren Abschnitt des Ganglion geniculatum laterale. Es präsentiert sich in der Abbildung 6 als ein kürzeres Gebilde, bei welchem durch eine Markzwischenschicht scheinbar ein kleiner lateraler Abschnitt abgesondert wird und das wiederum nach oben eine aus feineren Markfasern bestehende und dicht am inneren Rand des Nucleus rotundus verlaufende Bahn absendet. Dieser Faserzug scheint in einem Kern zu enden, der unmittelbar medial und etwas dorsal von dieser Schnittebene des Nucleus rotundus zu liegen kommt. Diese Bahn möchte ich als Tractus geniculo-thalamicus (tr. gen. thal.) und den fraglichen, mit diesem Faserzug in Verbindung stehenden Kern vorläufig mit dem Buchstaben G. bezeichnen. Unmittelbar medial vom Ganglion geniculatum laterale sehen wir die Commissura transversa Gudden (co. tr. G.), die sich jetzt in zwei Abschnitte teilt, einen grösseren dorsalen (p. dors.) und einen schmächtigeren, ventralen (p. ventr.). Die pars dorsalis scheint sich in einem rundlichen und in der Abbildung als eine Kernmasse sich präsentierenden Gebilde aufzusplittern. Die Abbildung 7 illustriert die Verhältnisse etwas weiter caudal von der oben beschriebenen Schnittebene. Das Ganglion geniculatum laterale, sowie auch der Nucleus rotundus sind nicht mehr deutlich zu erkennen. Dagegen treten in der Bildfläche der Praetectalkern (nucl.praet.) und der laterale Abschnitt des spiriformen Kerns (nu.spir.l.v.) auf. Ganz medial und ventral sehen wir den Querschnitt der Oculomotoriuswurzel (N. III) und unmittelbar dorsal und medial vom Tractus opticus, den Tractus opticus basalis (tr. b. o.), der auch eine deutliche Lichtung zeigt, was auf die Kernnähe dieses Opticusbündels deutet. Von diesem war bis jetzt noch nicht die Rede. Er lässt sich als ein dem übrigen Tractus opticus medial anliegendes und von ihm im Aussehen etwas abgesondertes Bündel erst in einem etwas caudal vom Nucleus rotundus gelegenen Schnitte erkenen und ist an der Schnittfläche dorsal von der Oculomotoriuswurzel zu finden. In der Abbildung 7 sehen wir unmittelbar dorsal vom Nucleus basalis opticus die Umrisse eines Kernes, der unmittelbar unter dem Tractus strio-thalamicus (tr. str.-thal.) zu liegen kommt und sich auch lateralwärts in der Form eines Dop-

pelkerns erstreckt. Diesen Kern betrachte ich als Endstätte der Commissura transversa Gudden (nu. co. tr. G.). In der Abbildung 9 sehen wir ein Nisslbild der entsprechenden Schnittebene. Hier präsentiert sich dieses Gebilde als ein Doppelkern (nu. co. tr. G.) mit einer kleinen Markzwischenschicht. Die Abbildung 11 gibt uns das gleiche Gebilde bei starker Vergrösserung wieder. Wir sehen aus diesem, dass der mediale Abschnitt des Kernes aus dichten, kleineren und grösseren polygonalen Ganglienzellen, zwischen denen ziemlich zahlreiche Gliaelemente zerstreut sind, und der laterale Abschnitt aus wenigen, dicht gelegenen, grösseren und kleineren Zellelementen besteht. Der mediale Kernabschnitt entsendet nach oben ein dichtes Faserbündel, das unmittelbar lateral vom Tractus strio-thalamicus zu liegen kommt und an der Schnittebene in seiner optimalen Länge getroffen wird. Oben mündet es in einen grossen ovalen Kern, dem von CRAIGIE und BRICKNER ('27) der Name des ovalen Kerns (nu. ov.) gegeben wurde. Die Verfolgung dieses in Wirklichkeit nicht ganz vertikal, sondern von oben nach unten etwas caudalwärts ziehenden Faserzuges, läszt keinen Zweifel an seinen engeren Beziehungen zum medialen Abschnitt des von uns als Nucleus commissura transversa bezeichneten Kernes. Ich möchte diese Bahn als Tractus nuclei commissurae transversae α (tr. nu. co. tr. α) bezeichnen. Ausserdem möchte ich besonders hervorheben, dass diese Bahn ganz bestimmt weder zu einem Mamillarkörper noch zum Nucleus basalis opticus Beziehungen hat. Der letztgenannte Kern ist in der Ausgangsebene dieses Faserbündels gar nicht zu sehen, da er viel caudaler liegt. Der laterale Abschnitt des Commissuralkerns sendet ebenfalls eine breite aber aus wenig dicken und dichten Fasern bestehende und stark gebogene "S"-förmige Bahn, die nach oben lateralwärts verläuft und dorsal und medial vom Nucleus praetectalis in einem Kern endigt. Diesen Kern möchte ich mit dem Nucleus spiriformis medialis dorsalis identifizieren. Diese Bahn tritt erst in den caudalen Ebenen besonders stark zum Vorschein, wenn Tractus α schon aus der Schnittfläche verschwunden ist. Das weist darauf hin, dass der laterale Abschnitt des Commissuralkerns etwas caudaler liegt als der mediale Kern. In der gleichen Ebene tritt der Nucleus basalis opticus unmittelbar dorsal und etwas lateral von dem Oculomotoriuswurzelquerschnitt und unmittelbar ventral vom mächtigen Tractus strio-thalamicus, medial an den letzten Ausläufern des Tractus opticus anliegend, hervor. In der Ebene des Austritts der Oculomotoriuswurzel hat der basale Opticuskern seine grösste Ausdehnung. Im Nisslbild sieht man grosse, ziemlich spärliche Ganglienzellen, zwischen denen kleinere Zellelemente zerstreut sind. Dieses gibt dem Kern ein charakteristisches Aussehen, was ihn bestimmt vor Verwechslung mit Nucleus commissurae transversae wahrt.

Die Befunde BECCARI's, in Bezug auf den basalen Opticuskern und dessen Verbindungen beim Lacertaembryo sind auch bei den Vögeln nachweisbar, sogar in einer ausgesprochenen Weise wie bei den Reptilien. So demonstriert uns die Abbildung 8 diesen sehr gut entwickelten Kern

(nu. b. o.), der einen mächtigen Faserzug (tr. nu. b. o. ad. nu. III) zum Nucleus III sendet.

Auch die Verbindung mit dem Interstitialkern Cajal's, im Sinne BECCARI's Befunde bei Lacerta muralis, ist im Vogelmittelhirn deutlich nachweisbar.

Wenn wir unsere Feststellungen bei den oben beschriebenen thalamischen Kernen zusammenfassen, so kann man diese mit folgenden Worten charakterisieren: Corpus geniculatum laterale, Nucleus commissurae transversae und der basale Opticuskern liegen räumlich in der nächsten Nachbarschaft von einander. An den Frontalschnitten erscheint zuerst das Corpus geniculatum laterale, dann, unmittelbar an ihm anliegend und etwas medio-caudal davon ist der Doppelkern der Commissura Gudden sichtbar. Direkt basal und etwas caudal liegt der Nucleus basalis opticus, der eine starke Entwicklung aufweist. Diese topographischen Beziehungen sind auf Frontalschnitten nicht immer sofort einleuchtend, sie werden aber auf horizontalen und sagitalen Schnittserien völlig bestätigt.

Diese drei Kerne sehen wir in der Abbildung 3 am Sagitalschnitt in obenerwähnter Reihenfolge liegen, wobei auch der Tractus α des nucleus commissurae transversae Gudden sehr gut zu sehen ist.

Der Sagitalschnitt der Abbildung 1 demonstriert uns das zweiteilige Corpus geniculatum laterale (g. g. l.), dem der Nucleus commissurae transversae Gudden (n. co. G.) caudal anliegt und die früher erwähnte S-förmige Bahn (tr. nu. c. o. tr. β) zum medialen spiriformen Kern (nu. sp. m. d.) sendet.

In der Abbildung 2 sehen wir eine andere Sagitalebene der gleichen Serie. Hier ist der Tractus (tr. nu. co. tr. α) zum Ovoidalkern (nu. ov.) deutlich dargestellt. Ausserdem sehen wir beim Ausgangspunkt dieses Faserzuges den basalen Opticuskern (nu. b. o.). Ganz klar aber werden diese topographischen Verhältnisse an der Abbildung 4 demonstriert. Hier sehen wir auf dem Horizontalschnitt diese drei Gebilde in obenerwähnter Reihenfolge dicht neben einander liegen. Ganz frontal ist das Ganglion geniculatum laterale (g. g. l.), das die zwei früher erwähnte Abschnitte aufweist, zu sehen. An ihn grenzt der längliche Nucleus commissurae transversae, in welchem wir anhand dieses Bildes einen fronto-medialen (nu. co. tr. G. p. fr. m.) und einen postero-lateralen Abschnitt (nu. co. tr. G. p. p. l.) unterscheiden können. Ausserdem ist die diesem Kern zuströmende Commissura transversa (co. tr. G.) zu sehen. Ganz caudal liegt der durch sein Aussehen so charakteristische basale Opticuskern (nu. b. o.), in dem die zugehörende basale Opticuswurzel (tr. b. o.) mündet. Auf der rechten Seite ist der Schnitt etwas tiefer getroffen, sodass die basale Opticuswurzel und deren Kern isoliert zur Darstellung gebracht werden. Anhand dieses Schnittes werden alle Feststellungen über diese Kerne an Frontalschnitten besonders beweisend; erstens die Einheitlichkeit dieses Doppelkerns der Commissura transversa Gudden, zweitens die räumlichen Beziehungen zwischen beiden thalamischen Verbindungen desselben, d.h.

die fronto-mediale Lage des Tractus α und die caudo-laterale Lage des Tractus β. Anhand dieses Schnittes werden auch die Abbildungen 1 und 2 verständlicher. So sehen wir z.B. die S-förmige Bahn (tr. β) und ihren Kern neben dem Corpus geniculatum laterale, wobei dieser laterale Kern in der Abbildung 1 neben dem durch die Marklamelle abgesonderten kleineren Abschnitt des Ganglion geniculatum laterale zu liegen kommt. Diese letzte Nachbarschaft ist aus der Abbildung 4 ohne weiteres verständlich. Ausserdem ist die Schnittführung leicht zu verstehen, wodurch an der Abbildung 1 die S-förmige Bahn isoliert dargestellt wird: diese Schnittführung ist nicht nur sagital und paramedian, sondern auch schräg, d.h. die pars fronto-medialis des Nucleus commissurae transversae und der basale Opticuskern bleiben vom Schnitt verschont und medial von diesem liegen. An der Abbildung 2 werden der isoliert dargestellte Tractus α und der Nucleus opticus basalis durch eine paramediane, aber axial gerichtete Schnittführung, bei der die pars postero-lateralis und Ganglion geniculatum laterale ausserhalb des Schnittes bleiben, sichtbar.

Ich glaube durch diese Untersuchung den Kern der Commissura transversa Gudden und seine topographischen Beziehungen zum Ganglion geniculatum laterale und zum basalen optischen Kern im Vogelgehirn nachgewiesen zu haben. Es liegt dann die Annahme sehr nahe, dass dieser Kern als das bis jetzt im Vogelgehirn vermisste Ganglion geniculatum mediale anzusehen ist. Seine Lage in Bezug auf das Ganglion geniculatum laterale (dicht anliegend, etwas medio-caudal davon) unterstützt diese Annahme vollkommen. Auch seine doppelkernige Struktur spricht sehr dafür.

Eine kurze Uebersicht der Literatur ergibt, dass von den älteren Autoren nur BELLONCI ('88) einen annähernd ähnlichen Befund in Bezug auf die Lokalisation des Kernes der Commissura transversa Gudden erhoben hat. So schreibt er über den Verlauf der Commissura transversa (inferior): „Sie liegt hinter und ein wenig über dem Tractus opticus, begleitet dessen hintere Wurzel auf ihrem Verlaufe nach hinten und endigt im hinteren Teil des Mittelhirns in der Nachbarschaft des Nucleus peduncularis."

Der Kern der letzteren wurde aber von ihm nicht genau angegeben und er spricht auch nicht über ein Ganglion geniculatum mediale in dem Abschnitt über das Vogelgehirn. Die späteren Autoren, bis auf die neuesten Untersuchungen von HUBER und CROSBY und von GRAIGIE, haben auch keine vollkommene Klarheit in dieser Frage gebracht. Speziell wurde von niemandem bis jetzt die engen topographischen Beziehungen zwischen diesen drei Kernen erwähnt. EDINGER und WALLENBERG ('99) haben den Nucleus lateralis als Kern der Decussatio inferior angesehen. Sie haben diese seitlich am Thalamusrand oberhalb des Ganglion geniculatum laterale lokalisiert. Ueber ein Corpus geniculatum mediale sprechen sie nicht. EDINGER ('08) gibt ein Ganglion geniculatum mediale an, seitlich am Thalamus in der Höhe des Nucleus anterior thalami und oberhalb des Nucleus rotundus, ohne diesen Kern näher zu beschreiben. Diese Annahme

wurde später von keinem Autor mehr unterstützt und scheint mir zu wenig begründet zu sein. KAPPERS ('21), in Uebereinstimmung mit MESDAG, will den Nucleus semilunaris als Kern der Commissura transversa ansehen. Ein Corpus geniculatum mediale wurde von ihm im Vogelgehirn nicht angegeben. GROEBBELS ('24) gibt keinen bestimmten Kern als solchen der Commissura transversa an. Es wird von ihm kein Ganglion geniculatum mediale erwähnt. RENDAHL ('24) beschreibt in seiner gründlichen embryologischen Arbeit keinen bestimmten Kern als Kern der Commissura transversa oder als Ganglion geniculatum mediale. Dabei hat er aber zweifellos diesen Kern gesehen, aber in anderer weise gedeutet. So scheint es mir, dass der Nucleus 2 *b*, respektive der Nucleus superficialis internus, der als ein breiter, länglicher, aus ziemlich grossen vorwiegend polygonalen Zellen bestehender Kern aus demselben Mutterboden wie das Corpus geniculatum laterale sich herausdifferenziert, unserem Kern der Commissura transversa Gudden entspricht, zum mindesten einem Teil desselben. Andererseits scheint mir der Nucleus externus RENDAHL auch zum Kern der Commissura Gudden zu gehören, speziell dem caudo-lateralen Abschnitt desselben. Alle anderen Autoren, die die Existenz des Nucleus externus annehmen, scheinen mir im gleichen Irrtum zu sein. Uebrigens bezieht sich das auch auf GROEBBELS ('24), der den Kern der Commissura Gudden zweifellos gesehen hat, aber einerseits als Corpus geniculatum laterale internum, andererseits als Mamillare gedeutet hat. Bei zweiteiliger Struktur unseres Ganglion geniculatum mediale ist eine solche Verwechslung zu verstehen. Daraus folgt, dass aus seiner Vierer-Gruppe die zwei übrigen Kerne, d.h. geniculatum laterale externum und geniculatum laterale tertium posterius, zum eigentlichen geniculatum laterale gehören. GRAIGIE ('28) scheint unseren Kern teilweise gesehen zu haben, weil er diesen mit dem Nucleus superficialis internus-RENDAHL möglicherweise identifizieren will. Andererseits meinte CRAIGIE, dass diese medial, vom geniculatum laterale sich befindende Zellmasse teilweise dem Nucleus entopeduncularis, teilweise dem Nucleus intercalatus angehören könnte (l.c. Seite 398). HUBER und CROSBY ('29) scheinen einen Teil des Nucleus der Commissura transversa Gudden richtig erkannt zu haben. Teilweise haben sie aber auf dessen Kosten die Existenz des Nucleus externus RENDAHL bestätigt, teilweise die Dimensionen des basalen Opticuskerns vergrössert. Ueber die Existenz eines Corpus geniculatum mediale sprechen sie nicht, sowie auch im allgemeinen nicht über die sonst klaren Beziehungen zwischen diesen von mir untersuchten Kernen.

Von den Verbindungen des Nucleus commissurae transversae Gudden mit den thalamischen Kernen, die ich oben beschrieben habe, ist der Tractus *a* zum Nucleus ovoidalis schon längst beobachtet aber unrichtig und sehr verschieden gedeutet worden. EDINGER und WALLENBERG ('98) haben diesen Faserzug als Tractus thalamo-mamillaris bezeichnet, in der Vermutung, dass dies eine dem Fasciculus Vicq d'Azyr analoge Verbindung sei. Ein eigentlicher Mamillarkörper wurde von diesen Autoren indessen

nicht nachgewiesen. EDINGER ('08) wiederholte die Deutung dieses Faserzuges im gleichen Sinn. GROEBBELS gab wieder einen Tractus thalamomamillaris an, wobei er den gleichen Faserzug meinte. Ich habe schon auseinandergesetzt, wie er den Nucleus commissurae transversae als Nucleus mamillaris deutete, sodass als sehr wahrscheinlich anzunehmen ist, dass er diese Bahn bis zum wirklichen Kern verfolgt, aber ihn nicht als Commisuralkern, sondern als Nucleus mamillaris gedeutet hat. RENDAHL ('24) zweifelte sehr an der Deutung Edinger's in Bezug auf diesen Tractus, da er einen Mamillarkörper von ihm als nicht nachgewiesen hielt. Die späteren Autoren wie CRAIGIE ('28), HUBER und CROSBY ('29) beschrieben diesen Faserzug als Tractus nuclei ovoidalis, ohne die andere Endstätte dieser Bahn mit Sicherheit angeben zu können.

Die von mir beschriebene zweite Bahn aus dem caudolateralen Stück des Kerns der Gudden'schen Commissur zum Nucleus spiriformis medialis dorsalis habe ich nur von HUBER und CROSBY ('29) als Tractus tectothalamicus dorsalis in der Abbildung 34 angegeben gefunden, ohne dass diese Bahn im Text besprochen wurde, sonst hat keiner von den Autoren in den von mir gelesenen Arbeiten diese Bahn beschrieben und gedeutet.

Die Verbindungen des Corpus geniculatum laterale waren bis jetzt unter dem von EDINGER geprägten Namen, des Stylus des Corpus geniculatum, bekannt. Wie aus vorliegender Untersuchung folgt, kann man mindestens zwei stärkere Faserzüge, die von dem Corpus geniculatum ausgehen, annehmen. Der von mir beschriebene Tractus geniculo-frontalis wurde bereits von HUBER und CROSBY ('29) als Bestandteil des Tractus thalamofrontalis intermedius angegeben.

Die caudal gelegene interthalamische Verbindung des Corpus geniculatum laterale ist möglicherweise mit dem von GROEBBELS angegebenen Stylus corporis geniculati lateralis externi identisch. Nur scheint es mir, dass der mit diesem in Verbindung stehende thalamische Kern nicht der Nucleus spiriformis laterale ventralis ist, sondern ein anderer, mehr frontal und medial gelegener Kern. Die Natur dieses Kernes ist mir nicht mit voller Sicherheit bekannt, es ist nur über ihn zu sagen, dass er in der Höhe des Nucleus ovoidalis liegt und in diesen caudalwärts fast fliessend übergeht. Ich bezeichne ihn vorläufig mit dem Buchstaben G. und hoffe, dass spätere Untersuchungen Klarheit darüber verschaffen werden.

Ich betrachte es als selbstverständlich zu betonen, dass mit der Beschreibung des Kernes der Commissura transversa Gudden, die Frage ihrer caudalen Verbindungen sicher nicht erschöpft ist. In meiner letzten Arbeit habe ich besonders betont, dass die Verbindung dieser Commissur mit dem Corpus posticum nicht von mir verfolgt und studiert wurde, da das Material dafür nicht geeignet war. Dasselbe möchte ich auch jetzt betonen und besonders hervorheben, dass die vorliegende Untersuchung sich lediglich auf das Studium des Commissuralkerns allein beschränkte.

Der Vergleich der topographischen Verhältnisse der von mir untersuchten Kerne im Thalamus des Vogelgehirns mit analogen Befunden im

Thalamus der Reptilien, ergibt eine grosse Aehnlichkeit der Beziehungen. Das topographische Verhalten dieser Gebilde im Thalamus der Reptilien, welches von mir am Schluss meiner früheren Arbeit hervorgehoben wurde, ist im Prinzip das gleiche wie bei den Vögeln. KAPPERS erwähnt in seinem Werk ('21), dass einige thalamische Kerne im Vogelgehirn unter dem Druck der stark entwickelten Hemisphären gegen die Basis wandern, so z.B. das Corpus geniculatum laterale. Aehnliches können wir bei unseren drei untersuchten Kernen feststellen. Sie sind im Vergleich mit ihrer Lage im Reptilienthalamus alle gegen die Basis gewandert, ohne in axialer Richtung ihr relatives topographisches Verhältnis verloren zu haben und sind nur enger aneinander gepresst worden.

Die entsprechenden intrathalamischen Verbindungen dieser Kerne, die bei Reptilien horizontal verlaufen, haben jetzt wegen dieser Verdrängung eine vertikale Lage angenommen.

Der Vergleich zwischen den entsprechenden Gebilden in beiden Tierklassen in Bezug auf die weitere Evolution derselben, ergibt die Tatsache, dass die beiden Corpora geniculata ihre weitere Entwicklung bekommen haben durch gewisse Komplizierung der Struktur und Vermehrung der Verbindungen. Im Gegensatz dazu scheint der basale Opticuskern im Prinzip der gleiche geblieben zu sein. Das weist meines Erachtens darauf hin, dass seine Herkunft primärer und archaischer ist als die der anderen Kerne. Als Beweis dafür dient auch die Erscheinung, dass in der aufsteigenden Tierreihe der basale Opticuskern allmählich verschwindet und nur in seltenen Fällen als ein rudimentäres Gebilde nachweisbar ist, während die beiden Geniculata eine mächtige Evolution durchmachen [1]).

ABKÜRZUNGEN:

nu. sp. m. d. = nucleus spiriformis medialis dorsalis; nu. dors. lat. ant. = nucleus dorsalis lateralis anterior; nu. rot. = nucleus rotundus; nu. co. tr. G. = nucleus commissurae transversae Gudden; nu. ov. = nucleus ovoidalis; n. b. o. = nucleus basalis opticus; tr. opt. = tractus opticus; g. g. l. = ganglion geniculatum laterale; tr. nu. co. tr. α = tractus nuclei commissurae transversae α; tr. nu. co. tr. β = tractus nuclei commissurae transversae β; p. p. l. = pars posterior lateralis; p. fr. m. = pars fronto-medialis; tr. gen. fr. = tractus geniculo-frontalis; dec. opt. = decussatio optica; nu. praet. = nucleus praetectatlis; n. III = nucleus III; co. su. inf. = commissura supra-infundibularis; tr. gen. thal. = tractus geniculo-thalamicus; co. ant. = commissura anterior; co. M. = commissura Meynert; p. dors. = pars dorsalis; p. ventr. = pars ventralis; tr. str. thalam. = tractus strio-thalamicus; nu. rub. = nucleus ruber; brach. conj. ant. = brachium conjunctivum anterius; tr. nu. b. o. ad. nu. III = tractus nuclei basalis optici ad nucleum III; undul. = undulatus; dom. = domesticus.

---

[1]) Ich benütze diese Gelegenheit hinzuweisen auf einen Fehler in meiner vorigen Arbeit S. 223 bei der Zitierung von BECCARI's Artikel „Il centro tegmentale o interstiziale ed altre formazioni poco note nel mesencefalo e nel diencefalo di un rettile (1923), wo ich den Kern ζ (zeta) seiner Fig. 11 und 12 als γ (gamma) bezeichnete. BECCARI vermutet die Anlage des Corp. geniculatum mediale der Reptilen in seinem Kern δ (delta).

Abb. 1
Melopsittacus undul.
(Sagit. W.-P.)

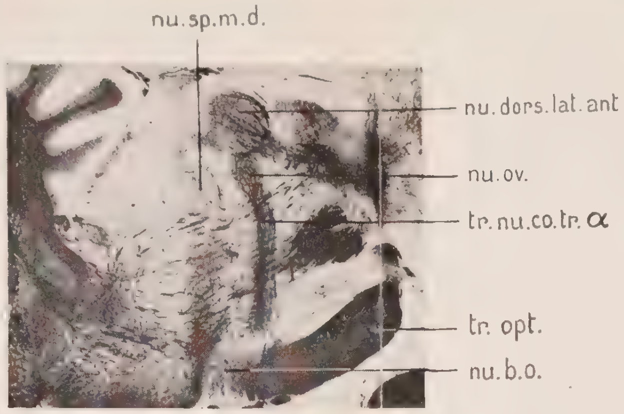


Abb. 2
Melopsittacus undul.
(Sagit. W.-P.)

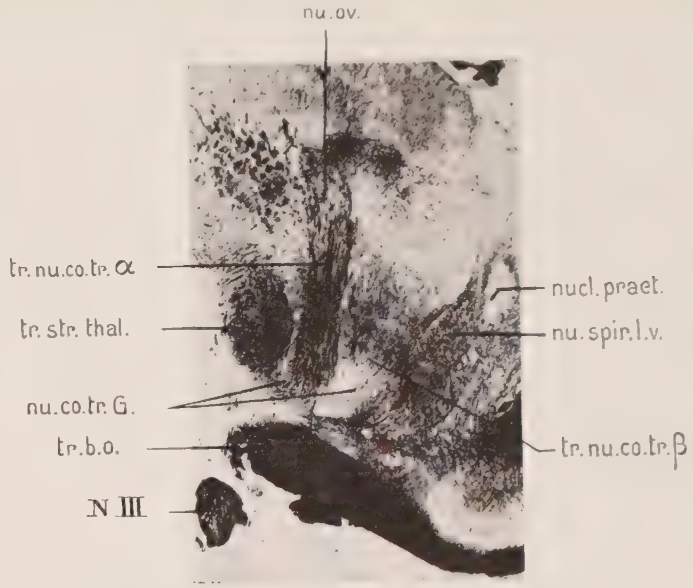


Abb. 7
Melopsittacus undul.
(front. W.-P.)

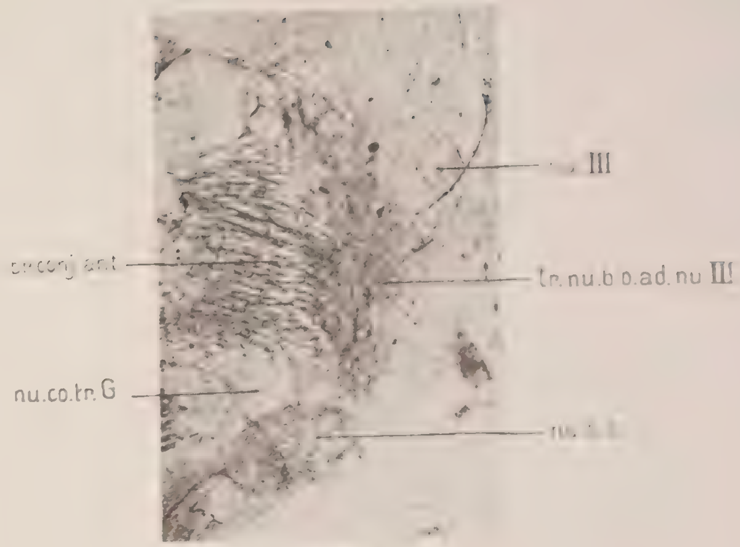


Abb. 8
Passer dom.

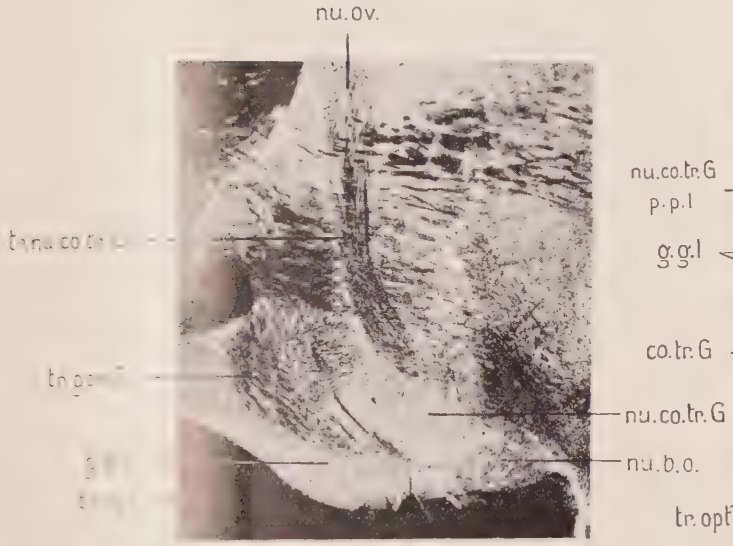


Abb. 3
Passer dom.
(Sag. W.-P.)

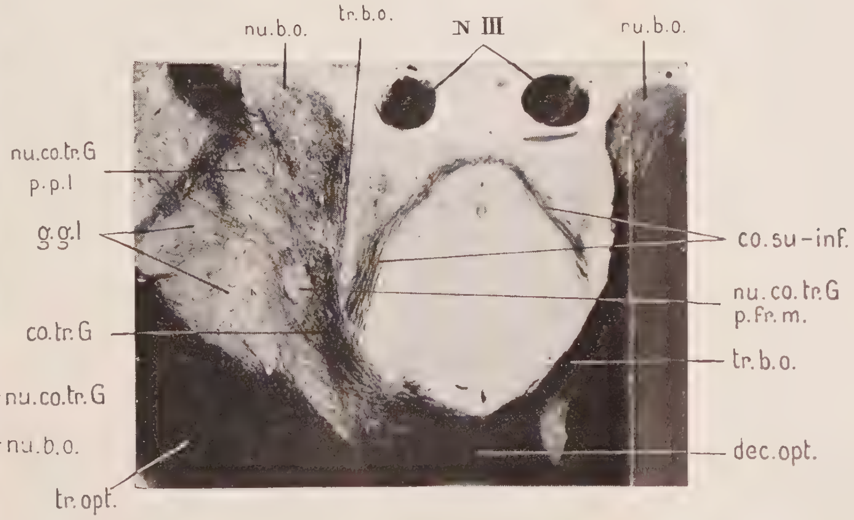


Abb. 4
Columba dom.
(horiz. W.-P.)

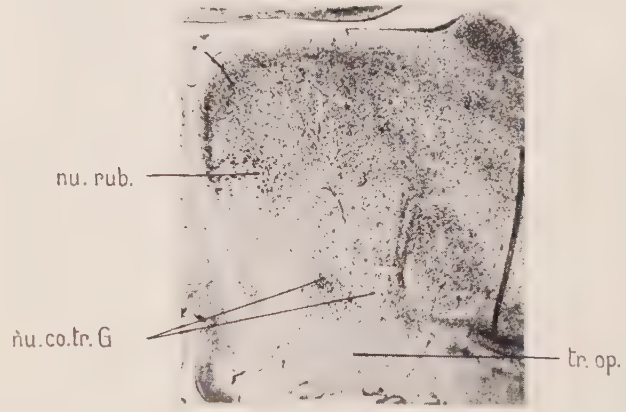


Abb. 9
Columba dom.
(front. Nissl)

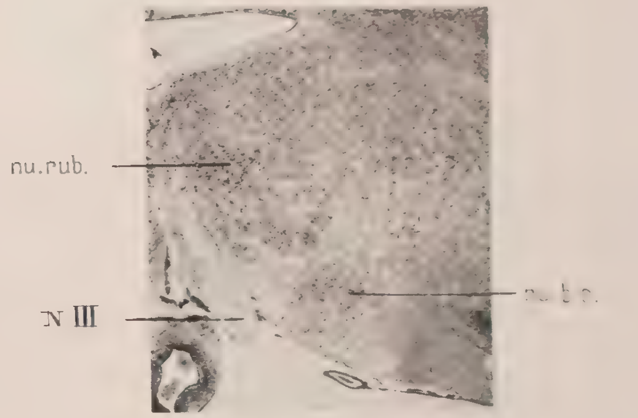


Abb. 10
Columba dom.

Abb. 5
Passer dom.
(front. W.-P.)

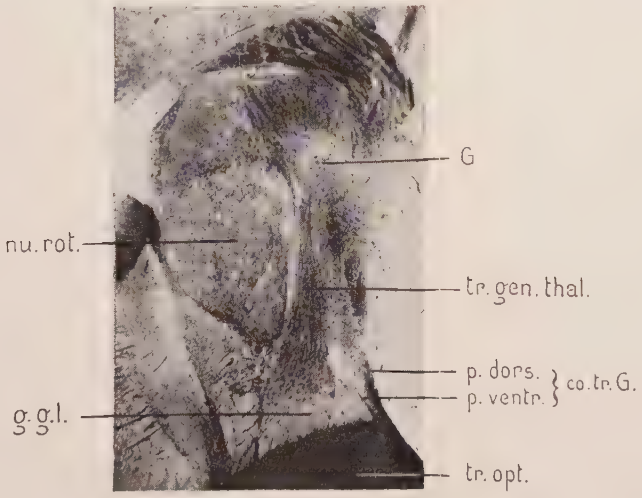


Abb. 6
Passer dom.
(front. W.-P.)

Abb. 11
Columba dom.
(front. Nissl)

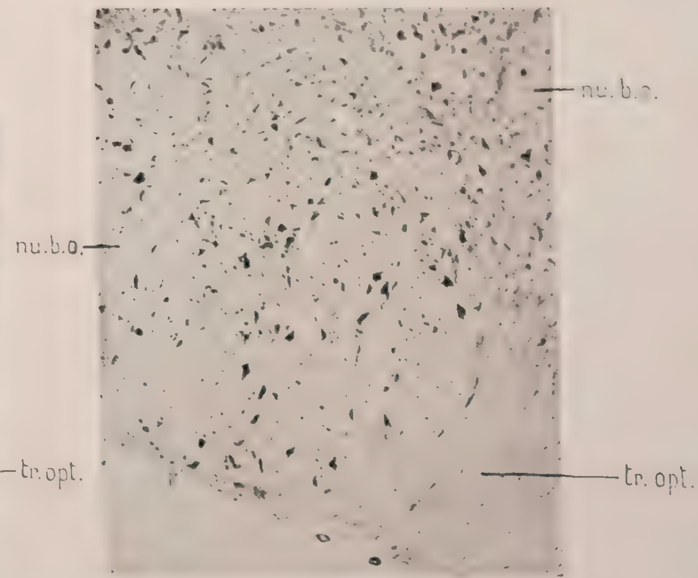


Abb. 12
Columba dom.
(front. Nissl)

LITERATURVERZEICHNIS.

ARIËNS KAPPERS, C. U. 1920—1921. Die vergleichende Anatomie des Nervensystems der Wirbeltiere und des Menschen. Bohn, Haarlem.

BELLONCI, J. 1888. Ueber die centrale Endigung des Nervus opticus bei den Vertebraten. Zeitschr. f. wissensch. Zool., Bd. **47**, S. 1—46.

BUMM. 1883. Das Groszhirn der Vögel. Zeitschr. f. wissenschaft. Zool., Bd. **38**, S. 430—467.

CRAIGIE, E. HORNE. 1928. Observations on the brain of the humming bird (Chrysolampis mosquitus Linn. and Chlorostilbon caribaeus Lawr.) Jour. Comp. Neur., vol. **45**, no. 2, pp. 377—483.

CRAIGIE, E. HORNE and BRICKNER, R. M. 1927. Structural parallelism in the midbrain and tweenbrain of teleosts and of birds. Proc. Kon. Akad. v. Wetens. te Amsterdam, vol. **30**, pp. 695—704.

EDINGER, L. 1908. Vorlesungen über den Bau der nervösen Zentralorgane des Menschen und der Tiere. Siebente Auflage, Bd. **2**, Leipzig.

EDINGER, L. und WALLENBERG, A. 1899. Untersuchungen über das Gehirn der Tauben. Anat. Anz., Bd. **15**, S. 245—271.

EDINGER, L., WALLENBERG, A. und HOLMES, G. 1903. Untersuchungen über die vergleichende Anatomie des Gehirns. 5. Das Vorderhirn der Vögel. Abhandl. der Senckenbergischen Naturf. Gesellsch., Bd. **20**, Heft 4, S. 343—426.

FREY, E. 1933. Über die basale Opticuswurzel und die caudalen Verbindungen der Commissura transversa Gudden der Reptilien. Proc. Kon. Akad. v. Wetens. te Amsterdam, vol. **36**, S. 217.

GROEBBELS, FRANZ. 1924. Untersuchungen über den Thalamus and das Mittelhirn der Vögel. Anat. Anz., Bd. **57**, S. 385—415.

HUBER, CARL G. and CROSBY, ELIZABETH CAROLINE. 1929. The nuclei and fiber paths of the avian diencephalon etc. The Journal of comparative neurology, vol. **48**, no. 1.

MESDAG, T. M. 1909. Bijdrage tot de ontwikkelingsgeschiedenis van de structuur der hersenen bij het kipembryo. Dissertation, Groningen.

MÜNZER, E. und WIENER, H. 1898. Beiträge zur Anatomie und Physiologie des Centralnervensystems der Taube. Monatschr. f. Psychiat. u. Neurol., Bd. **3**, S. 397—406.

RENDAHL, H. 1924. Embryologische und morphologische Studien über das Zwischenhirn beim Huhn. Acta. Zool., Bd. **5**, S. 241—244.

STIEDA, L. 1869. Studien über das zentrale Nervensystem der Vögel und Säugetiere, Zeitschrift f. Wiss. Zool., Bd. **19**.

STILLING, J. 1882. Untersuchungen über den Bau der optischen Centralorgane.

WALLENBERG, A. 1898 a. Die secundäre Acusticusbahn der Taube. Anat. Anz., Bd. **14**, S. 353—369. 1898 c. Das mediale Opticusbündel der Taube. Neurol. Centralbl., Jahrg. **17**, S. 532—537. 1904. Neue Untersuchungen über den Hirnstamm der Taube. Anat. Anz., Bd. **24**, S. 142—155, S. 357—369.

Physiology. — *Recherches sur l'allongement du pied de l'escargot (Helix pomatia L.) après une contraction en réponse d'un stimulant électrique.* (Communication provisoire). Par N. POSTMA. (Communicated by Prof. H. J. JORDAN.)

(Communicated at the meeting of March 25, 1933).

Introduction: L'école de physiologie comparée du Prof. JORDAN à Utrecht s'est occupée de l'étude du tonus des muscles lisses. Néanmoins elle s'est surtout limitée aux études des phénomènes d'allongement, qui se manifestent, après un repos bien défini, chez le pied excisé d'escargot, qui n'a pas été stimulé auparavant.

Si l'on compose un diagramme de l'allongement et du temps $\left(\frac{dl}{dt}\right)$ on obtient une courbe du genre de celle que reproduit la fig. 1.

JORDAN a interprété cette courbe de la manière suivante: La partie ver-

Fig. 1. Courbes d'allongement d'un pied d'escargot, qui suivent, après un repos défini, la préparation. Au bout déchargement complèt pour mesurer la tension élastique.

I. Objet, dont les ganglions sont exstirpés. Chargement de 20 gr. Déchargement partiel, restant 10 gr.

II. Objet, dont les ganglions sont exstirpés.

III. Objet. dont les ganglions pédieux sont encore présents.

ticale, qui se trouve au commencement de la courbe, représente l'établissement de l'équilibre entre la tension élastique et la charge. — Les muscles striés des vertebrés présentent au début le même phénomène. Bientôt après cependant l'allongement cesse, le muscle s'opposant à un allongement indéfini. Ici la courbe se prolonge horizontalement, sous forme d'un asymptote. — Le pied de l'escargot au contraire continue à céder au poids. L'allongement est rendu possible par le fait que les micelles du tissu musculaire se disjoignent conformement aux lois de la plasticité. Pendant l'allongement ces micelles se rapprochent en direction transversale, et par conséquent le frottement interne augmente; c'est pourquoi l'inclinaison de la courbe diminue. Les raisons principales, qui ont conduit JORDAN à admettre que la résistance contre l'allongement est de nature plastique, résulte des faits suivants:

1. que l'allongement du pied d'escargot en repos est irréversible et qu'il se continue même après déchargement partiel (fig. 1, courbe I);
2. l'influence spécifique des ganglions pédieux sur le tonus viscosoïde. A la figure 1 sont reproduites deux courbes qui se sectionnent (II et III). La différence mutuelle des deux courbes doit être attribuée au fait que chez un des deux objets (III) les ganglions pédieux sont encore présents, alors qu'ils ont été enlevés à l'autre (II). On voit que la présence des ganglions provoque d'abord une inclinaison plus brusque et qu'après la courbe tend vers l'horizontale. JORDAN attribue donc à l'intervention des ganglions pédieux une diminution du tonus viscosoïde. Après le „coude" (→+) de la courbe le tonus se renforce par rapport à celui du pied à ganglions exstirpés. Le pied se trouve alors dans l'état d'un „tonus central". Maintenant la contraction musculaire chez le pied d'escargot dépend du ganglion cérébral; les ganglions pédieux n'influencent ni la contraction, ni la descente de la courbe de contraction. C'est pourquoi JORDAN conclut que les courbes décrites répresentent des phénomènes spécifiques du tonus viscosoïde.
3. que l'inclinaison de la courbe augmente au fur et à mesure qu'on élève la température du pied, le frottement interne allant en diminution.

Pour cette raison JORDAN *fait une distinction entre le tonus „viscosoïde" (intrafibrilaire) des muscles lisses* **en repos**, *le tétanotonus et la viscosité* (A. V. HILL) *(interfibrilaire) qui se rencontrent chez tous les muscles* ***en activité.***

Le pied une fois allongé, il n'était pas possible de rétablir la position primitive au moyen d'une contraction provoquée par un stimulant électrique. Après la contraction et le relâchement la courbe reprend immédiatement son allure primitive comme si l'excitation n'avait pas eu lieu. La courbe de relâchement n'avait pas encore été étudiée séparément.

DE MAREES VAN SWINDEREN (1927) et JORDAN (1930) ont obtenu une rétraction du pied, donnant une courbe totalement différente de celle qui vient d'être décrite dans l'alinéa précédant. Ils ont nommé ce phénomène:

la contraction tonique. Pour obtenir cette réaction ils ont soumis l'objet à des variations rapides de température. Cette situation artificielle ne se présentant pas dans la vie normale de l'animal, le problème de la reprise de l'état tonique original reste à résoudre.

Dans un travail de BOZLER (1930 p. 596) on trouve: „Wenn man den Muskel (l'adducteur de Pecten)[1]) entlastet, ihn zur Kontraktion bringt und ihn dann einige Minuten in Ruhe lässt, ist der Dehnungswiderstand wieder wie zuerst und man kann den Versuch wiederholen". Bienque BOZLER ne donne aucune figure, qui nous permette de nous rendre compte du fait que la courbe est réversible, il eut été cependant désirable d'étudier la phase descend du pied d'escargot après la contraction libre.

Nous nous sommes proposés d'étudier les phénomènes de l'allongement après contraction, afin de comparer ceux-ci avec ce qui se passe dans le pied préparé comme il sera décrit ci-dessous.

Dans le premier cas nous sommes en présence d'un processus de restitution, occasionnant un relâchement en principe indépendant de la charge, mais qui ne se manifeste que sous l'influence du poids. Dans le second cas nous avons affaire à un allongement causé par le poids, mais pour lequel, d'après JORDAN, du moins pour la durée et la température en question il ne se présente un relâchement pareil.

Les deux formes fournissent des graphiques semblables, parfois même entièrement identiques. La forme même de la courbe ne peut donc jamais nous renseigner sur ce qui se passe dans le muscle. Seule la réaction du muscle, pendant son allongement, sur les différentes facteurs à l'action desquels on le soumet, pourra nous informer à ce sujet (voir aussi JORDAN 1933).

Le but de notre travail est uniquement d'étudier le rôle de la température sur l'allongement après excitation. Dans une publication ultérieure nous espérons pouvoir communiquer nos résultats concernant l'influence d'autres facteurs sur cette courbe.

Préparation: La coquille brisée et éloignée, on détache, d'un coup de ciseaux, la bourse intestinale. Au besoin on écarte de la même manière les ganglions. Pour monter l'objet sur la tablette de l'appareil de DE MAREES VAN SWINDEREN on fixe au moyen d'une épingle, courbée en forme de S et munie d'un fil, le bout distal du pied. A son tour le fil est serré entre un pincefil. Alors on accroche également le bout proximal du pied, passe le fil par le frein (1, fig. 3) et l'on enroule le file autour du disque (4, fig. 3), destiné à indiquer l'étendue de l'allongement.

Après un repos, p. e. d'une demi-heure, on charge d'un poids le crochet situé au bout libre du fil (3, fig. 3), et l'allongement commence.

Pour mon travail je me suis servi de l'appareil décrit par DE MAREES

[1]) Remarquez que ce muscle a une toute autre fonction.

VAN SWINDEREN (1927; voir aussi JORDAN—HIRSCH 1927 ou JORDAN—V. D. FEEN 1929) avec les modifications suivantes:

Afin de faciliter le refroidissement désiré au moyen de morceaux de glace le cylindre en verre, destiné à contenir de l'eau a été remplacé par un

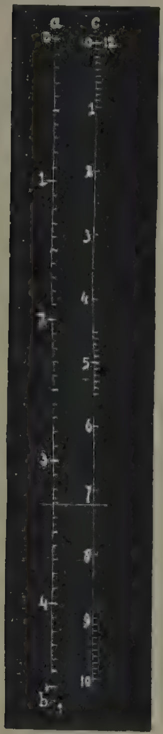


Fig. 4.

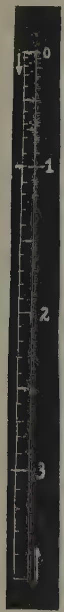


Fig. 2

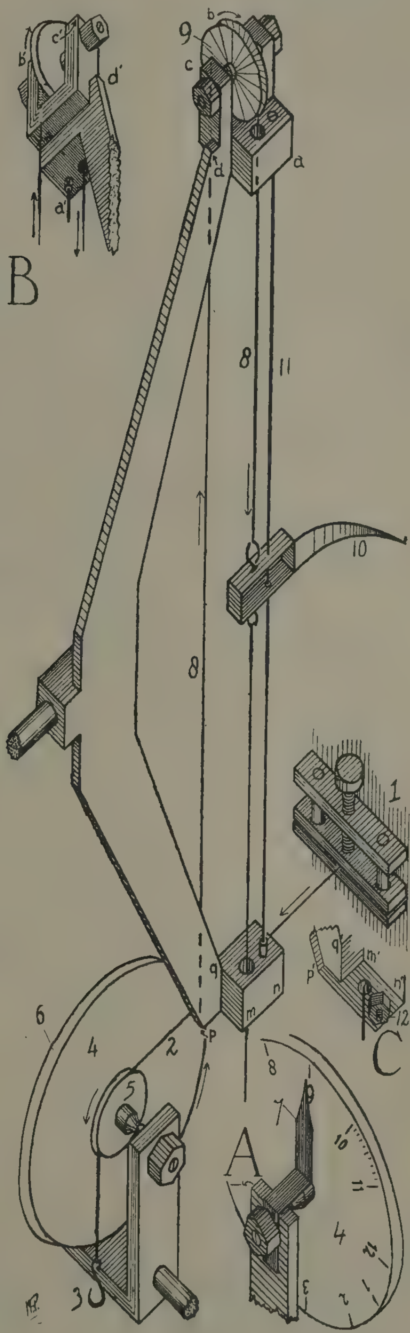


Fig. 3

Fig. 2. Contrôle de l'aggrandissement de l'inscripteur de KEITH-LUCAS.

Fig. 3. L'inscripteur de KEITH-LUCAS modifié. 1. Le frein de l'appareil de DE MAREES VAN SWINDEREN. 3. Crochet, porteur du poids. 4. Disque, destiné à indiquer l'étendue de l'allongement. 6. Jante du disque. 7. Style fixé. — Explication détaillée dans le texte.

Fig. 4. Contrôle de l'amplification uniforme du disque, enregistré au moyen de l'inscripteur modifié.

aquarium. Une ouverture dans deux de ses parois laisse passer le tube intérieur contenant le pied d'escargot ; un peu de plasticine bouchant le tout. Le bras portant le disque gradué a été supprimé, et le disque lui même est porté par un statif séparé.

L'allongement du pied de l'escargot est enregistré suivant la méthode de HERTER (1931a p. 715—717 et 1931b p. 263—264) au moyen d'un kymographe. Afin de pouvoir faire une comparaison quantitative des différentes courbes, il a fallu vérifier la valeur de l'aggrandissement réalisé pour chaque m. m. d'allongement, par l'instripteur de KEITH—LUCAS (LANGLEY, 1911). Le contrôle a appris que pour une amplification de 1¾ fois pour le 1er millimètre d'allongement, l'appareil de KEITH—LUCAS augmente constamment de rapport jusqu' à donner pour le 35e m. m. une amplification de $6\,^2/_3 \times$ (voir fig. 2). La correction comparative de cette erreur aurait considérablement compliqué les calculs. C'est pour cette raison que nous avons modifié l'appareil de KEITH—LUCAS (voir fig. 3).

Tout d'abord, pour éviter l'amplification démesurée produite par la transposition du mouvement tournant en mouvement rectiligne, nous avons fait disparaître le levier. Il fallait cependant enregistrer au moyen d'une amplification suffisante et rendre celle-ci aussi uniforme que possible. A cet effet la jante (6) du disque gradué (4) a été creusée en gouttière. Dans la gouttière se meut un fil dont un bout est fixé dans un point quelconque de la jante, tandisque l'autre bout du fil (8), ayant passé par une poulie (9) est fixé au levier d'inscripteur (10). L'amplification se déduit alors du rapport entre le rayon du disque et celui du galet (5) où passe le fil (2) porteur de la charge.

Nous communiquons ici les résultats de deux séries d'expériences. Pour chacune d'elle un seul animal a été utilisé.

DONNEÉS EXPÉRIMENTALES.

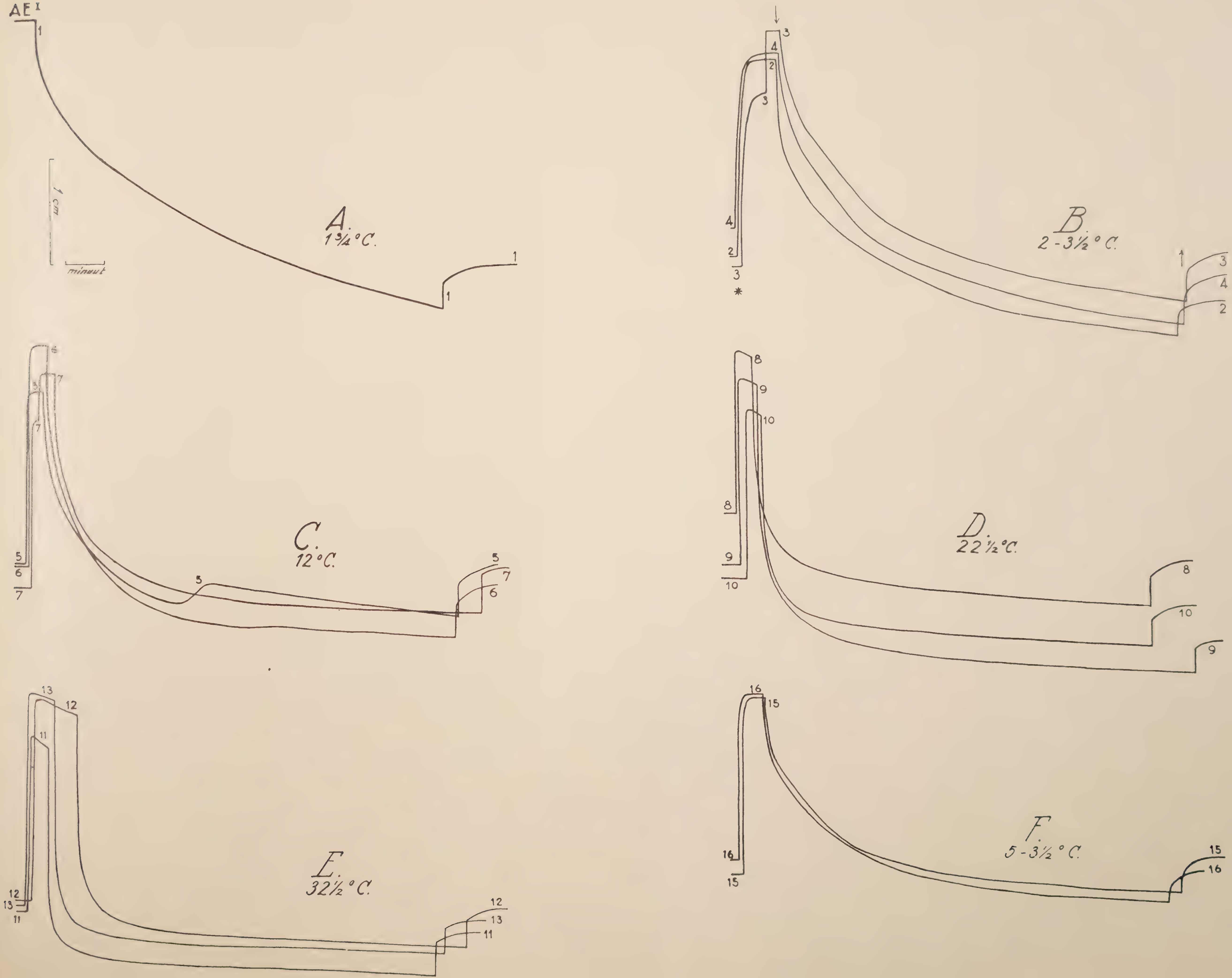
Objet AE I.

Poids 20 gr ; en liberté l'escargot atteint parfois une longueur de $7^1/_2$ c.m. Préparation le 15-VI-'32 à 16^{00} h., ganglions exstirpés ; le pied a été muni d'une charge de 5 gr. ; on ferme le frein ; longueur du pied $5^1/_2$ c.m. Le 16-VI-'32 à 10^{30}, température $1^3/_4$° C. 11^{45}. Commencement de l'allongement. On ouvre le frein (↓). Charge 20 gr. L'étirement se poursuit durant 10 min., après quoi la charge est enlevée (↑). Le pied est forcé de se contracter, par de courtes faradisations (*) (distance de la bobine secondaire 12—$10^1/_2$ c.m.)

1. Première	courbe	d'allongement	(fig. 5 A)
2, 3, 4	température	2—$3^1/_2$° C.	(„ B)
5, 6, 7	„	12° C.	(„ C)
8, 9, 10	„	$22^1/_2$° C.	(„ D)
11, 12, 13	„	$32^1/_2$° C.	(„ E)
15, 16	„	5—$3^1/_2$° C.	(„ F)
18, 19	„	45° C.	(„ 5 G)

Dans chaque série de 3 ou 2 expériences on observe pour la même température aussi bien une contraction qu'un allongement. Suit alors un repos d'une demi-heure au moins, pendant lequel la température est élevée au degré désiré. — Longueur du pied après la 19e expérience (frein fermé) : 7.8 c.m.

N. POSTMA : Recherches sur l'allongement du pied de l'escargot (Helix pomatia L.) après une contraction en réponse d'un stimulant électrique.
(Fig. 5 A, 5 B, 5 C, 5 D, 5 E, 5 F).



On constate donc que pour une même température les mêmes courbes

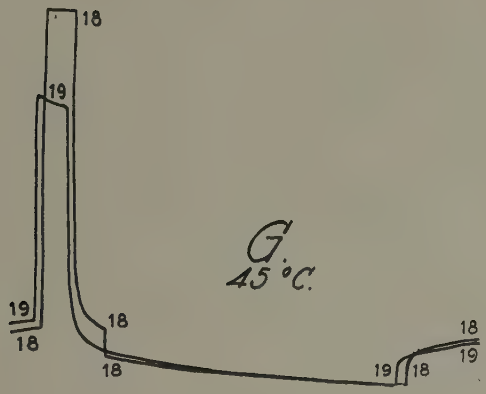


Fig. 5. Myogrammes des contractions d'un pied d'escargot par suite d'un stimulant électrique. Les ganglions ont été exstirpés. Voir explications dans le texte.

peuvent se répéter facilement, ce qui nous a permis de construire pour chaque série de 3 (2) expériences une courbe moyenne (fig. 6).

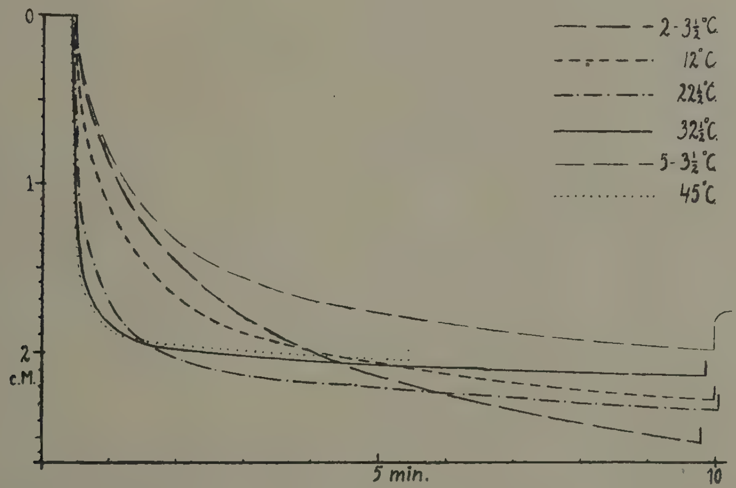


Fig. 6. Courbes moyennes de la phase descend, construites d'après celles de la figure 5.

Si maintenant nous comparons entre elles ces différentes myogrammes nous voyons que leur aspect général se modifie, à température ascendente, d'après une règle constante : la première décrescence s'accentue et devient plus brusque ; alors après un coude plus aigu au fur et à mesure que la

température monte, la courbe s'aplatit et tend vers une ligne de plus en plus horizontale.

L'interprétation de ces phénomènes doit, à notre avis, être cherchée du moins pendant la première partie de la descente, dans l'influence de la température sur le système visqueux du pied, permettant ainsi un étirement. Entretemps l'équilibre s'établit entre la tension des tissus et le poids de la charge ; seulement ici l'effet direct de cette influence symbolisée dans la courbe reste masqué par le fait que le pied continue à céder à l'allongement.

Après cela, au fur et à mesure que la température monte, la charge est portée plus tôt et à un niveau plus élevé à tel point même que les deux courbes se sectionnent mutuellement. Ceci ne peut donc pas ressortir de la seule influence de la température sur un système visqueux. Il faudra reconnaître ici en outre une influence favorable de la température sur les éléments contractiles eux-mêmes, donc sur le tétanotonus.

Pour confirmer cette façon de voir, nous possédons encore d'autres données, dont nous proposons de publier ultérieurement les détails. Nous ne voulons pas résoudre la question de savoir si seule l'élévation de la température suffit pour expliquer le renforcement du tétanotonus après stimulation ; ou si l'allongement plus rapide lui-même ou plus tôt les deux ensemble doivent entrer en ligne de compte.

La descente succédant à la contraction du muscle, non chargé, exprime donc deux fonctions différentes : dans la première partie c'est surtout la résistance visqueuse, pour la seconde un tétanotonus, qui semble jouer un rôle prépondérant. Le rôle de l'élément contractile est en proportion inverse de celui de la résistance visqueuse.

On aura remarqué que les courbes des deux objets, obtenues à la même température se sectionnent (voir fig. 1 * II/III) également par suite de la présence ou de l'absence respective des ganglions pédieux. Le même phénomène s'observe chez le pied sans ganglions, mais à température différente, lors de la descente après contraction. La cause en est qu'une température plus élevée affaiblit d'abord la viscosité (interfibrilaire ?) ; en même temps elle augmenterait la résistance par tétanotonus.

La courbe d'allongement du muscle en repos à différentes températures ne révèle que la résistance visqueuse.

On voit donc qu'il n'est pas permis de conclure du simple fait de la ressemblance des courbes d'allongement ou des contrastes correspondants (sectionnement des courbes) qu'on a à faire à une même série de phénomènes.

Peut-être nous objectera-t-on que le caractère typique des courbes doit être attribué au fait que les contractions se réalisent à une température variant d'une série d'expériences à l'autre, à savoir la température propre de l'allongement correspondant.

Pour éliminer cette difficulté, nous avons répété la série d'expériences en ayant soin de rammener après chaque allongement, la température à 10° C. et de la garder constante pendant un repos d'une demi-heure. De même, après la contraction la température désirée fut-elle rétablie, et laissa-t-on à l'objet le temps (une demi-heure) d'arriver à cette température, après quoi l'allongement commença.

Comme on peut voir à la figure 7 le résultat ne change pas au point de vue qualitatif. Au point de vue quantitatif les contrastes sont un peu plus faibles, ainsique la possibilité de répéter la courbe diminue.

Dans des publications ultérieures de nous-mêmes ainsi que du Prof. H. J. JORDAN seront communiqués les résultats de l'examen des rapports mutuels entre le tonus viscosoïde et le tétanotonus.

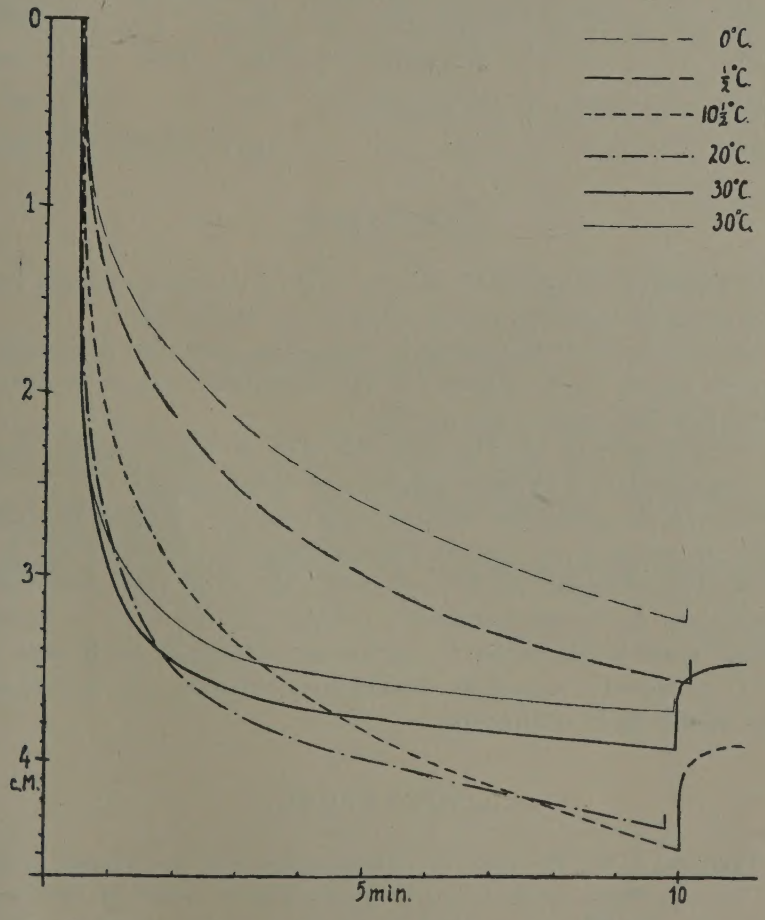


Fig. 7. Courbes correspondantes d'une autre série d'expériences, comme celles de la figure 5: la contraction s'effectue toujours à la même température (10° C.), qui ensuite est remplacée par la température indiquée ci-dessus.

DONNEÉS EXPÉRIMENTALES.

Objet AK VII

Poids 21 gr. Préparation faite le 25-I-'33 à 19 h.; ganglions exstirpés. Charge 5 gr., frein fermé. Commencement de l'allongement le 26-I-'33 à 10 h. avec une charge de 20 gr. Température: 0° C. Après chaque allongement (pendant 10 min.) la charge est enlevée. Contraction au moyen d'un stimilus électrique (faradisations): distance de la bobine secondaire: 10 c.m.

Température du repos	Commencement du repos	Fin et excitation du repos	Température du repos	Commencement du repos	Fin; chargement du repos	Commencement de l'allongement	Fin par charge écartée de l'allongement	Numération
10°	10^{20}	— 10^{50}	0°	10^{55}	— 11^{25}		11^{35}	2
10°	11^{40}	— 12^{10}	$\frac{1}{2}$°	12^{15}	— 12^{45}		12^{55}	3
10°	14^{30}	— 15^{00}	$10\frac{1}{2}$°	15^{00}	— 15^{35}		15^{45}	5
10°	17^{05}	— 17^{35}	20°	17^{40}	— 18^{10}		18^{20}	7
10°	18^{25}	— 19^{20}	30°	19^{25}	— 19^{55}		20^{05}	8
10°	20^{10}	— 20^{40}	30°	20^{45}	— 21^{15}		21^{25}	9

RÉSUMÉ.

L'inscripteur de KEITH—LUCAS a été modifié de façon à rendre l'amplification constante pendant tout le trajet de l'enregistrement.

La descente qu'on obtient après la contraction d'un pied d'escargot, non chargé, se répète facilement, et doit être considérée comme la résultante de deux fonctions différentes, à savoir:

1. en tout premier lieu la résistence d'un systême visqueux, ensuite
2. l'intervention d'éléments contractiles (tétanotonus).

Par moyen de stimulation électrique on peut donc rétablir la résistance tonique viscosoide à un état de raccoursissement plus élevé de ce qu'il était au commencement de la contraction. En outre nous avons prouvé contrairement à ce que supposent certains auteurs, que la résistance *prolongée* après la contraction va surtout aux dépens de la viscosité dynamique et en moindre mesure du tétanotonus. Il paraît, que ce tétanotonus soit un résidu de la contraction.

LITTERATURE:

1911. LANGLEY, J. N. The effect of various poisons upon the response to nervous stimuli chiefly in relation to the bladder. Journal of Physiology 43, p. 128.

1918. JORDAN, H. J. Über die Physiologie der Muskulatur und des zentralen Nervensystems bei hohlorganartigen Wirbellosen; insbesondere bei Schnecken. Ergebnisse der Physiologie 16, p. 87—227.

1927. DE MAREES VAN SWINDEREN, J. W. De regeling van den tonus in de spieren van Helix pomatia L. Thèse de doctorat Utrecht.

JORDAN, H. J. (en collaboration avec HIRSCH, C. G.) Übungen aus der vergleichenden Physiologie. SPRINGER, Berlin.

1929. JORDAN, H. J. et V. D. FEEN Jr., P. J. Methoden und Technik der Nerven- und Muskelphysiologie bei wirbellosen Tieren. Abderhaldens Handbuch der biologischen Arbeitsmethoden Abt. IX, Teil 4. URBAN & SCHWARZENBERG, Berlin.

JORDAN, H. J. Allgemeine vergleichende Physiologie der Tiere. DE GRUYTER & Co., Berlin.

1930. BOZLER, E. Untersuchungen zur Physiologie der Tonusmuskeln. Zs. f. vergl. Physiol. 12, 579—602.

JORDAN, H. J. Der Tonus glatter Muskeln als Funktion der Muskelfluidität. Tonus, tonische Kontraktion, Tonus im Verhältnis zum Tetanus. Proc. Akad. v. Wetensch. Amsterdam 33, II, N°. 8, 788—791.

1931*a*. HERTER, K. Untersuchungen über den Muskeltonus des Schneckenfusses. Zs. f. vergl. Physiol. 13, 709—739.

b. ——— Der Jordansche „Halbtierversuch". Zs. f. v. Ph. 15, 261—308.

1933. JORDAN, H. J. Plastischer Tonus und „Reiztonuseffekt" bei „hohlorganartigen" Tieren. Archivio d. Sc. biologiche 18, 191.

Laboratory of comparative Physiology.

Utrecht, Februari 1933.

CORRIGENDUM.

In der von Professor Dr. M. W. WOERDEMAN angebotenen Abhandlung „*Rückbildung der Schneidezähne im Ober- und Unterkiefer*" — vd. die „Proceedings", Vol. XXXV, Nr. 9 1932 — lese man die Unterschriften von Tafelabbildung 1 und 3 wie folgt:

Reduktion des einen, Agenesie des anderen mittleren Schneidezahnes.

TH. E. DE JONGE-COHEN.